Topology 3

Problem Set 6 — H. Reich/F. Lenhardt
SS 2013 — Due: -

The exercises on this sheet are voluntary; they will not be corrected and should not be handed in.

Exercise 1

(i) Let $f\colon A \to B$ be a cofibration. Show that applying $\operatorname{map}(-, X)$ to the cofiber sequence $A \to B \to \operatorname{cof}(f)$ yields a fibre sequence.

(ii) Suppose $f\colon \mathbb{E} \to \mathbb{F}$ is a map of spectra such that each f_n is a fibration. Define $\operatorname{fib}(f)$ levelwise and prove that there is an exact sequence

$$\cdots \to \pi_{k+1}\mathbb{F} \to \pi_k \operatorname{fib}(f) \to \pi_k\mathbb{E} \to \pi_k\mathbb{F} \to \dots$$

(iii) Show that the functors $\pi_k \operatorname{map}(-, \mathbb{E})$ for a fixed spectrum $\mathbb{E}$ and $k \in \mathbb{Z}$ give rise to a long exact sequence when applied to a cofiber sequence.

Exercise 2
Let B be a pointed space. Suppose that for some $l \geq 2$ there is a weak homotopy equivalence $f\colon B \to \Omega^l B$. Use this to define a spectrum $\mathbb{E}_B$ with $\pi_k(\mathbb{E}_B) \cong \pi_k(\mathbb{E}_B)$ for all $k \in \mathbb{Z}$.