Freie Universität Berlin

Topology 3

Problem Set 4
SS 2013

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Exercise 1
Let $Y = \{0\} \cup \{n^{-1} \mid n \in \mathbb{N}\} \subset \mathbb{R}$ equipped with the subspace topology. Determine a CW-approximation of Y, i.e., a CW-complex X together with a weak equivalence $f\colon X \to Y$. Show that f cannot be a homotopy equivalence.

Exercise 2
For a differentiable manifold M let $p\colon TM \to M$ denote the tangent bundle. Let $s_0\colon M \to TM$ be the zero section and denote by $p_0\colon EM \to M$ the restriction of p to $EM = TM - s_0(M)$. Set $N = O(n)$, $M = GL(n, \mathbb{R})$ and let $i\colon N \to M$ denote the inclusion. Study the connectivity of the natural map $Ei\colon EN \to EM$.

Exercise 3
Construct a free $\mathbb{Z}/3\mathbb{Z}$-operation on S^∞. Show that there exists a path connected space X with $\pi_1(X) \cong \mathbb{Z}/3\mathbb{Z}$ and $\pi_k(X) \cong 0$ for all $k \geq 2$.

Exercise 4
Compute all homotopy groups of $\Omega\mathbb{R}P^\infty$.