Topology 3

Problem Set 3 | H. Reich/F. Lenhardt
SS 2013 | Due: 19.6.

Exercise 1
Show that $\pi_n(\mathbb{R}P^n, \mathbb{R}P^{n-1})$ is different from $\pi_n(\mathbb{R}P^n/\mathbb{R}P^{n-1})$ for $n \geq 2$. You may use that $\pi_n(S^n) \cong \mathbb{Z}$.

Exercise 2
Let $f\colon X \to Y$ be a continuous pointed map of pointed spaces. Let $Z(f)$ be the mapping cylinder of f. Show that there is a long exact sequence

$$\cdots \to \pi_n(X) \to \pi_n(Y) \to \pi_n(Z(f), X) \to \pi_{n-1}(X) \to \dots$$

Exercise 3
Let $f\colon X \to Y$ be a continuous pointed map of pointed spaces. For a point $x \in X$, let $\eta_X(x)$ be the path $t \mapsto (x,t)$ in $X \times [0,1]$ which descends to a loop in ΣX. In this way, we obtain a pointed continuous map $\eta_X\colon X \to \Omega\Sigma X$. Consider the diagram

$$\begin{array}{ccccc}
\mathrm{hfib(f)} & \longrightarrow & X & \longrightarrow & Y \\
 & & \downarrow{\scriptstyle \eta_X} & & \downarrow{\scriptstyle \eta_Y} \\
\Omega\,\mathrm{hcof(f)} & \longrightarrow & \Omega\Sigma X & \longrightarrow & \Omega\Sigma Y
\end{array}$$

where hfib(f) is the homotopy fiber of f and hcof(f) is the homotopy cofiber of f.
Construct a pointed map hfib(f) $\to$ hcof(f) making the diagram commutative up to homotopy.

Exercise 4
Let $f\colon S^1 \vee S^1 \to S^1 \times S^1$ be the canonical inclusion. Compute for all $n \in \mathbb{N}$ the map

$$[\Sigma^n f, S^1]\colon [\Sigma^n(S^1 \times S^1), S^1] \to [\Sigma^n(S^1 \vee S^1), S^1]$$