Homotopy Theory

Problem Set 2 — H. Reich/F. Lenhardt

SS 2013 — Due: 12.6.

Exercise 1
Let Z be a space.

(i) Let $p\colon E \to B$ be a Hurewicz fibration. Then also $p_*\colon \mathrm{map}(Z, E) \to \mathrm{map}(Z, B)$ is a Hurewicz fibration.

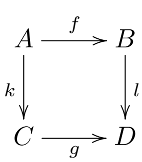
(ii) Let $i\colon A \to X$ be a cofibration. Then $i^*\colon \mathrm{map}(X, Z) \to \mathrm{map}(A, Z)$ is a Hurewicz fibration.

Exercise 2
Let $A \subset X$ be spaces and $a_0 \in A$. Check that the sequence of pointed sets

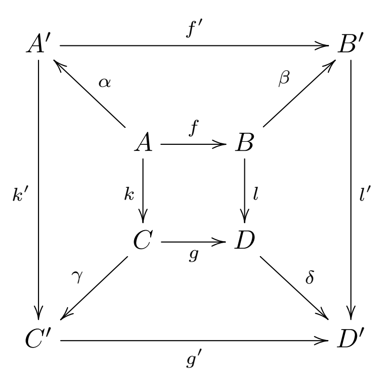
$$\pi_1(A, a_0) \to \pi_1(X, a_0) \to \pi_1(X, A, a_0) \to \pi_0(A, a_0) \to \pi_1(X, a_0)$$

is exact. Recall that a sequence of pointed sets $(M, m_0) \xrightarrow{f} (N, n_0) \xrightarrow{g} (P, p_0)$ is exact at (N, n_0) if the preimage of p_0 under g equals the image of f.

Exercise 3
We have seen that any map $f\colon A \to B$ has a factorization $f = \pi(f) \circ h(f)$ with $h(f)\colon A \to N(f)$ a homotopy equivalence and $\pi(f)\colon N(f) \to B$ a Hurewicz fibration. Show that for a commutative diagram



we can find a commutative diagram



with α, β, γ and δ homotopy equivalences and f', g', k' and l' Hurewicz fibrations.

Please turn the page!

Exercise 4
Show that for $n \geq 2$, we have

$$\pi_n(X \vee Y) \cong \pi_n(X) \oplus \pi_n(Y) \oplus \pi_{n+1}(X \times Y, X \vee Y)$$