Topology 2

Problem Set 13 H. Reich/F. Lenhardt
WS 2012/13 Due: 30.01.2013

Exercise 1
Show that there exists an isomorphism

$$\mathrm{Tor}_n^R(\bigoplus_{i\in I} M_i, N) \cong \bigoplus_{i\in I} \mathrm{Tor}_n^R(M, N).$$

Exercise 2
Let $M \to I^\bullet$ and $N \to J^\bullet$ be injective resolutions of the R-modules M and N. Show that

$$\begin{aligned}[I^\bullet, J^\bullet] &\to \mathrm{Hom}_R(M, N)\\ f &\mapsto H^0(f)\end{aligned}$$

is an isomorphism.

Exercise 3
Compute $\mathrm{Ext}_n^{\mathbb{Z}}(M, \mathbb{Z}/p\mathbb{Z})$ for $M = \mathbb{Z}, \mathbb{Z}[\frac{1}{p}]$ and $\mathbb{Z}[\frac{1}{p}]/\mathbb{Z}$ and all $n \in \mathbb{Z}$.
Hint: There is an exact sequence

$$0 \to \bigoplus_{i=0}^{\infty} \mathbb{Z} \xrightarrow{f} \bigoplus_{i=0}^{\infty} \mathbb{Z} \to \mathbb{Z}[\frac{1}{p}] \to 0$$

with $f((x_i)_{i\in\mathbb{N}}) = (x_i - px_{i-1})_{i\in\mathbb{N}}$, where $x_{-1} = 0$.

Exercise 4
Prove or disprove: There exists a topological space X such that

$$H^1(X; \mathbb{Z}) \cong \mathbb{Q}.$$