

## Topology 2

Problem Set 10  
WS 2012/13

H. Reich/F. Lenhardt  
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### Exercise 1

Let  $sk_k(\Delta^n)$  be the  $k$ -skeleton of  $\Delta^n$  with its usual  $\Delta$ -complex structure. Compute  $H_*(sk_k(\Delta^n))$ .

### Exercise 2

The non-orientable surface  $N_g$  of genus  $g$  has a  $CW$ -structure with one zero-cell,  $g$  1-cells and a 2-cell attached along the word  $a_1^2 a_2^2 \cdots a_g^2$ . Compute  $H_*(N_g)$ .

### Exercise 3

For a  $\mathbb{Z}$ -module  $M$  we set

$$t(M) = \{x \in M \mid \exists n \in \mathbb{N} \text{ with } nx = 0\}.$$

- (i) Show that  $t(-)$  is an additive functor  $t: \mathbb{Z}\text{-MOD} \rightarrow \mathbb{Z}\text{-MOD}$ , i.e.  $t$  is a functor and there is a natural isomorphism  $t(M \oplus N) \cong t(M) \oplus t(N)$ .
- (ii) Let  $0 \rightarrow L \rightarrow M \rightarrow N \rightarrow 0$  be a short exact sequence in  $\mathbb{Z}\text{-MOD}$ . Discuss at which spots the sequence  $0 \rightarrow t(L) \rightarrow t(M) \rightarrow t(N) \rightarrow 0$  obtained by applying the functor  $t$  is always exact. Find examples showing it is not always exact at the other spots.

### Exercise 4

Merry Christmas: 5 free points!