Topology 2

Problem Set 8 H. Reich/F. Lenhardt
WS 2012/13 Due: 12.12.2012

Exercise 1
Let P, Q be complex polynomials. We identify S^2 with $\mathbb{C} \cup \{\infty\}$.

(i) Show that there is a continuous map $f \colon S^2 \to S^2$ such that $f(z) = \frac{P(z)}{Q(z)}$ for all $z \in \mathbb{C}$ with $Q(z) \neq 0$.

(ii) Compute $\deg(f)$.

Exercise 2
Let X be a space.

(i) Show that $H_k(X \times S^n, X \times \{x_0\}) \cong H_{k-1}(X \times S^{n-1}, X \times \{x_0\})$ for all $k \in \mathbb{Z}$, $n \geq 1$ and $x_0 \in S^{n-1} \subset S^n$.

(ii) Show that $H_k(X \times S^n) \cong H_k(X) \oplus H_k(X \times S^n, X \times \{x_0\})$ for all $k \in \mathbb{Z}$, $n \geq 0$ and $x_0 \in S^n$.

(iii) Deduce that $H_k(X \times S^n) \cong H_k(X) \oplus H_{k-n}(X)$ for all $k \in \mathbb{Z}$ and $n \geq 0$.

Exercise 3
The real projective plane $\mathbb{R}P^2$ is homeomorphic to the space obtained by glueing a Moebius band M and a 2-disk D along their common boundary S^1. Determine the groups and homomorphisms in the Mayer-Vietoris sequence obtained by the decomposition of $\mathbb{R}P^2$ into M and D.

Exercise 4
Let X, Y be spaces with $(X_1, X_2, \dots, X_n)$ an open cover of X and $(Y_1, Y_2, \dots, Y_n)$ an open cover of Y. Let $f \colon X \to Y$ be a map such that $f(X_i) \subset Y_i$ and such that for each nonempty subset A of $\{1, 2, \dots, n\}$, the restriction of f $\bigcap_{a \in A} X_a \to \bigcap_{a \in A} Y_a$ induces an isomorphism on homology. Show that also $H_*(f) \colon H_*(X) \to H_*(Y)$ is an isomorphism.