Topology 2

Problem Set 7 — H. Reich/F. Lenhardt
WS 2012/13 — Due: 05.12.2012

Exercise 1
Compute $H_*(\Sigma_g)$ by considering the decomposition $\Sigma_g = X_1 \cup X_2$. Here Σ_g is the surface of genus g, decomposed as follows:

Hint: $\Sigma_g \setminus \mathring{D}^2 \simeq \bigvee_{i=1}^{2g} S^1$.

Exercise 2
Let $K \subset \mathbb{R}^3$ be a tame knot, for example

Compute $H_1(\mathbb{R}^3 \setminus K)$.
Hint: Use that there is a tubular neighborhood of K in $\mathbb{R}^3$, i.e. there is an open subset $N \subset \mathbb{R}^3$ with $K \subset N$ such that

$$(N, N \setminus K) \cong (\mathring{D}^2 \times S^1, (\mathring{D}^2 \setminus \{0\}) \times S^1)$$

Exercise 3
Consider a commutative diagram of abelian groups

$$\begin{array}{ccccccccccc} 0 & \longrightarrow & K & \xrightarrow{i} & A & \xrightarrow{\alpha} & B & \xrightarrow{p} & C & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle k} & & \downarrow{\scriptstyle a} & & \downarrow{\scriptstyle b} & & \downarrow{\scriptstyle c} & & \\ 0 & \longrightarrow & K' & \xrightarrow[i']{} & A' & \xrightarrow[\alpha']{} & B' & \xrightarrow[p']{} & C' & \longrightarrow & 0. \end{array}$$

where both rows are exact. Furthermore assume that k is surjective and c injective. Show that

$$0 \longrightarrow \ker(k) \xrightarrow{f} A \xrightarrow{g} A' \oplus B \xrightarrow{h} B' \xrightarrow{j} \operatorname{coker}(c) \longrightarrow 0$$

is exact, where $f(x) = i(x)$, $g(y) = (a(y), -\alpha(y))$, $h(u, v) = \alpha'(u) + b(v)$ and $j(w) = p'(w) + \operatorname{im}(c)$.

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Exercise 4
Let X be an n-dimensional CW-complex, i.e. we have

$$X_n = X_{n+1} = X_{n+2} = \dots$$

Show that $H_k(X) = 0$ for $k > n$.