Topology 2

Problem Set 5 H. Reich/F. Lenhardt
WS 2012/13 Due: 21.11.2012

Exercise 1
Let $C = (C_n, c_n)$, $D = (D_n, d_n)$ and $E = (E_n, e_n)$ be chain complexes. Let $\mathrm{Hom}(C, D)$ be the set of all chain maps from C to D and define

$$\mathrm{NHom}(C, D) = \{f \in \mathrm{Hom}(C, D) \mid f \sim 0\}.$$

Show that:

(i) $\mathrm{Hom}(C, D)$ is an abelian group and $\mathrm{NHom}(C, D)$ is a subgroup. We define

$$[C, D] = \mathrm{Hom}(C, D)/\mathrm{NHom}(C, D).$$

(ii) $f \sim g$ if and only if $f - g \in \mathrm{NHom}(C, D)$.

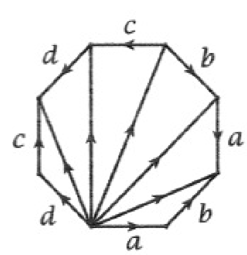
(iii) Composition induces a well-defined map

$$\begin{array}{ccc} [D, E] \times [C, D] & \to & [C, E] \\ ([k], [f]) & \mapsto & ([k \circ f]). \end{array}$$

Exercise 2
Let C be a chain complex of finite-dimensional vector spaces over some field k and assume that only finitely many C_i are non-zero. Show that

$$\sum_{i \in \mathbb{Z}} (-1)^i \dim_k(C_i) = \sum_{i \in \mathbb{Z}} (-1)^i \dim_k(H_i(C))$$

Exercise 3
Compute the Δ-homology of the surface of genus 2 with respect to the Δ-complex structure given by the following picture:



Please turn the page!

Exercise 4

Consider a commutative diagram of abelian groups and group homomorphisms

$$\begin{array}{ccccccccc} A & \xrightarrow{f} & B & \xrightarrow{g} & C & \xrightarrow{h} & D & \xrightarrow{j} & E \\ \downarrow{\scriptstyle a} & & \downarrow{\scriptstyle b} & & \downarrow{\scriptstyle c} & & \downarrow{\scriptstyle d} & & \downarrow{\scriptstyle e} \\ A' & \xrightarrow[f']{} & B' & \xrightarrow[g']{} & C' & \xrightarrow[h']{} & D' & \xrightarrow[j']{} & E'. \end{array}$$

such that both rows are exact.

(i) If b and d are monomorphisms and a is an epimorphism, then c is a monomorphism.

(ii) If b and d are epimorphisms and e is a Monomorphismus, then c is an epimorphism.

(iii) If a, b, d and e are isomorphisms, then c is also an isomorphism.