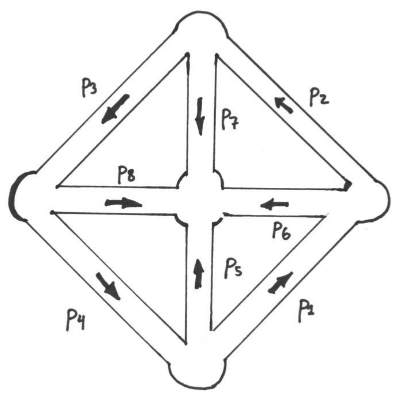
Exercises for

Topology 2

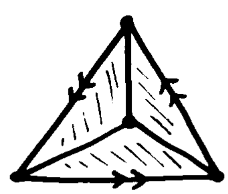
Problem Set 4 H. Reich/F. Lenhardt
WS 2012/13 Due: 14.11.2012

Exercise 1
We consider the following system of pipes, filled with some incompressible fluid (for example water).



Each arrow stands for a pump. For each pump, you can set its throughput, a real number. Positive numbers mean pumping in the direction of the arrow, negative numbers mean pumping in the other direction. If there is overpressure or low-pressure at any node, the system explodes. Find all pump settings for which the system does not explode.
How is this exercise related to the lecture?

Exercise 2
Let X be the Δ-complex arising from



by identifying the edges as indicated. Compute the Δ-complex homology of X. Can you find a Δ-complex Y such that $H_1^{\Delta}(Y) \cong \mathbb{Z}/n\mathbb{Z}$ for $n \geq 4$?

Exercise 3
We consider S^2 with the $\mathbb{Z}/2\mathbb{Z}$-action given by multiplication with -1. The quotient is $\mathbb{R}P^2$. Find a $\mathbb{Z}/2\mathbb{Z}$-equivariant Δ-complex structure on S^2. Describe a Δ-complex structure on $\mathbb{R}P^2$.

Please turn the page!

Exercise 4
Let $\mathcal{C}$ be a category and X, X' objects of $\mathcal{C}$. We consider the (covariant) functors

$$\begin{aligned} \mathrm{mor}_{\mathcal{C}}(X, -)&: \mathcal{C} \to \mathrm{SETS} \\ \mathrm{mor}_{\mathcal{C}}(X', -)&: \mathcal{C} \to \mathrm{SETS} \end{aligned}$$

For a morphism $f\colon X' \to X$ and any object Y of $\mathcal{C}$, we obtain a map

$$\begin{aligned} \eta(f)_Y : \mathrm{mor}_{\mathcal{C}}(X, Y) &\to \mathrm{mor}_{\mathcal{C}}(X', Y) \\ \phi &\mapsto \phi \circ f \end{aligned}$$

(i) Show that $\eta(f)$ defines a natural transformation from $\mathrm{mor}_{\mathcal{C}}(X, -)$ to $\mathrm{mor}_{\mathcal{C}}(X', -)$.

(ii) Show that each natural transformation $\eta\colon \mathrm{mor}_{\mathcal{C}}(X, -) \to \mathrm{mor}_{\mathcal{C}}(X', -)$ is of this form, i.e. there is $f\colon X' \to X$ such that $\eta = \eta(f)$.