Exercises for

Topology 2

Problem Set 3 — H. Reich/F. Lenhardt
WS 2012/13 — Due: 7.11.2012

Exercise 1
Let X be a CW-complex. Show that X is path-connected if and only if its 1-skeleton X_1 is path-connected.

Exercise 2
We consider $S^\infty = \bigcup_{n=0}^\infty S^n$ as a CW-complex with n-skeleton S^n. Show that each map $f\colon S^k \to S^\infty$ is homotopic to a constant map.

Exercise 3
Let $f\colon X \to Y$ be a continuous map.

(i) Show that continuous maps $g\colon M_f \to Z$ out of the mapping cylinder of f are in 1-1-correspondence with maps $\psi\colon X \to Z$ and $\phi : Y \to Z$ together with a homotopy $H\colon X \times [0,1] \to Z$ from $\psi \circ f$ to ϕ.

(ii) Define the mapping cone C_f of f as the space $M_f/j(X)$, where $j\colon X \to M_f$ denotes the closed embedding. Show that continuous maps from C_f to Z are in 1-1-correspondence with maps $\phi\colon Y \to Z$ together with a homotopy $H\colon X \times [0,1] \to Z$ from $\phi \circ f$ to a constant map.

(iii) Suppose $X = X_n$ is obtained from X_{n-1} by simultaneous attaching of n-cells via the attaching map ϕ. Use (ii) in order to describe continuous maps from X to Z in terms of maps from X_{n-1} to Z and certain homotopies.

Exercise 4
Let X and Y be CW-complexes and $f\colon X \to Y$ a cellular map, i.e. a continuous map sending the n-skeleton of X to the n-skeleton of Y for all n.

(i) Assume that X and Y are 1-dimensional. Describe a CW-structure on the mapping cylinder M_f of f. In fact, there is a natural CW-structure on M_f for arbitrary CW-complexes.

(ii) Now let X and Y be any finite CW-complexes. What is the Euler characteristic of M_f? Try not to use that M_f is homotopy equivalent to Y.