Topology 2

Homework 1 — H. Reich/F. Lenhardt
WS 2012/13 — Due: 24.10.2012

Exercise 1
A commutative diagram of vector spaces and linear maps of the form

$$\begin{array}{ccc} V_0 & \xrightarrow{f_1} & V_1 \\ {\scriptstyle f_2}\downarrow & & \downarrow{\scriptstyle F_2} \\ V_2 & \xrightarrow{F_1} & V_3 \end{array}$$

is a pushout diagram if it has the following universal property:
Is W a vector space together with linear maps $g_1 \colon V_1 \to W$, $g_2 \colon V_2 \to W$ such that $g_1 \circ f_1 = g_2 \circ f_2$, there is a unique linear map $g \colon V_3 \to W$ such that $g \circ F_1 = g_1$ and $g \circ F_2 = g_2$. Show that each diagram of vector spaces of the form

$$\begin{array}{ccc} V_0 & \xrightarrow{f_1} & V_1 \\ {\scriptstyle f_2}\downarrow & & \\ V_2 & & \end{array}$$

has a pushout, i.e. there is a vector space V_3 with linear maps $F_1 : V_2 \to V_3$, $F_2 : V_1 \to V_3$ such that

$$\begin{array}{ccc} V_0 & \xrightarrow{f_1} & V_1 \\ {\scriptstyle f_2}\downarrow & & \downarrow{\scriptstyle F_2} \\ V_2 & \xrightarrow{F_1} & V_3 \end{array}$$

is a pushout diagram.

Exercise 2
Let K_n be the complete graph on n vertices, i.e. the graph with n vertices where each two vertices are connected by an edge.

(i) Find an embedding of K_5 into the torus.

(ii) Find an embedding of K_6 into the torus.

(iii) Find an embedding of K_7 into the torus.