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Free Groups and Graphs

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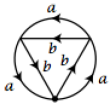
Homework 9 — Due: December 17, 2012

Problem 1

Denote by R_2 the graph with a single vertex v and 4 edges. Identify $\pi_1(R_2, v)$ with the free group with basis $\{a, b\}$. Consider the following covering X_H of R_2



and show that its corresponding subgroup $H \leq F(a, b)$ is normal. Furthermore, determine $F(a, b)/H$.

Problem 2

Find a pointed core graph (Γ, p) and an immersion $f\colon \Gamma \to R_2$ such that the image of $f_*\colon \pi_1(\Gamma, p) \to \pi_1(R_2, v) = F(a, b)$ is given by the subgroup $\langle aba^{-1}, aba^{-1}b^{-1}\rangle$.

Problem 3

Is $\{ab, ab^{-1}a, bcb^{-1}\}$ a basis of $F(a, b, c)$?

Problem 4

Find a basis of $\langle abba, ababa\rangle \cap \langle aba^{-1}, b\rangle \leq F(a, b)$.