Freie Universität Berlin
Institut für Mathematik

Prof. Dr. Elmar Vogt
Sebastian Meinert

Free Groups and Graphs

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Homework 12 Due: January 21, 2013

For a group G we denote its abelianization G/G' by G_{ab}. Here $G' \subseteq G$ is the commutator subgroup of G. Since any endomorphism $f\colon G \to G$ maps G' to G', f induces an endomorphism $f_{ab}\colon G_{ab} \to G_{ab}$ and we obtain a homomorphism

$$ab_G\colon End(G) \to End(G_{ab}),\ f \mapsto f_{ab}.$$

(In fact, ab is a functor from the category of groups to the category of abelian groups.) In Problem 1 of Homework 3 we proved that $(F_n)_{ab} \cong \mathbb{Z}^n$. We denote ab_{F_n} by ϕ_n.

Problem 1

a) Show that for any group G the homomorphism ab_G maps $Aut(G)$ to $Aut(G_{ab})$.

b) Show that $\phi_n^{-1}(Aut(\mathbb{Z}^n)) \subseteq End(F_n)$ contains for $n \geq 2$ the group $Aut(F_n)$ as a proper subgroup, i.e. $\phi_n^{-1}(Aut(\mathbb{Z}^n)) \setminus Aut(F_n) \neq \emptyset$.

Problem 2

List all Whitehead automorphisms of F_n which are in the kernel of ϕ_n.

Problem 3

a) Show that all inner automorphisms of F_n are in the kernel of ϕ_n (this holds, in fact, for any group G).

b) Show that for $n \geq 3$ there are automorphisms f of F_n in the kernel of ϕ_n which are not inner automorphisms of F_n.