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Free Groups and Graphs

Winter 2012/2013

Homework 11 Due: January 14, 2013

Problem 1

Explicitly describe the inverses of Whitehead automorphisms of F_n.

Problem 2

Let $\phi\colon F(a,b) \to F(a,b)$ be given by $\phi(a) = abbab$ and $\phi(b) = bababbab$.

(i) Show that ϕ is an automorphism.

(ii) Write ϕ^{-1} as a composition of Whitehead automorphisms.

Problem 3

Let $\{x_1, \dots, x_n\}$ be a basis of F_n. Consider the following automorphisms of F_n

P: $x_1 \mapsto x_2 \mapsto x_1$,

C: $x_1 \mapsto x_2 \mapsto \dots \mapsto x_n \mapsto x_1$,

I: $x_1 \mapsto x_1^{-1}$,

A: $x_1 \mapsto x_1 x_2$.

where any basis element not explicitly moved by an arrow "$\mapsto$" stays fixed. Show that P, C, I, and A generate $Aut(F_n)$ by showing that every Whitehead automorphism is a product of these automorphisms.