# FREIE UNIVERSITÄT BERLIN Institut für Mathematik



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## Free Groups and Graphs

Winter 2012/2013

Homework 10 Due: January 7, 2013

### Task 1

Find a basis of the subgroup  $\langle ab^{-1}a^{-1}, ba \rangle \cap \langle bba, aba^{-1}b^{-1}, aaab \rangle \leq F(a, b)$ .

### Task 2

Recall that the abelianization of  $F_2$  is given by  $\mathbb{Z}^2 = \mathbb{Z} \oplus \mathbb{Z}$ , and consider the natural map  $Aut(F_2) \to Aut(\mathbb{Z}^2)$ . Determine the images of all Whitehead automorphisms under this map and show that the map is surjective.

### Task 3

Let  $\varphi \in Aut(F(a,b))$  be given by  $\varphi(a) = a^{-1}ba$ ,  $\varphi(b) = bba$ . Write  $\varphi$  as a composition of Whitehead automorphisms.

#### Task 4

Have a Merry Christmas and a Happy New Year!