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Homotopy Theory

Summer 2015

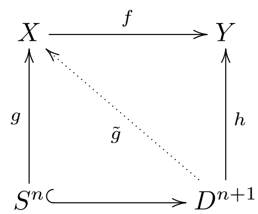
Homework 8 Due: June 11, 2015

Problem 15

Prove that the following statements are equivalent

(i) The map $f : X \to Y$ is a weak homotopy equivalence.

(ii) The image of f intersects every path component of Y and for every $n \geq 0$ in the following commutative diagram the dotted arrow can be found



such that $\tilde{g}$ extends g and $f \circ \tilde{g}$ is homotopic to h relative S^n.

Problem 16

Let $(X; X_1, X_2)$ be a CW-triad with $X = X_1 \cup X_2$. Abbreviate $X_0 := X_1 \cap X_2$ and let

$$M_{X_1,X_2} := X_1 \times \{0\} \cup X_0 \times [0,1] \cup X_2 \times \{1\} \subset X \times [0,1].$$

Let $\rho : M_{X_1,X_2} \to X$ be the restriction of the projection $X \times [0,1] \to X$ to M_{X_1,X_2}. Show that ρ is a homotopy equivalence.

(Please turn over.)

You may use or prove first the following fact. Let (X_1, X_0) be a CW-pair, and consider $M = X_0 \times [0,1] \cup X_1 \times \{1\} \subset X_1 \times [0,1]$ and

$$\begin{array}{ccc} & X_0 \times \{0\} & \\ {}^{i}\swarrow & & \searrow^{j} \\ M & \xrightarrow{\quad\rho\quad} & X_1 \end{array}$$

with ρ the restriction of the projection map $X_1 \times [0,1] \to X_1$ and $j(x,0) = x$ for $x \in X_0 \subset X_1$. Then there exist $\eta : X_1 \to M$ such that $i = \eta \circ j$, and homotopies h rel $i(X_0 \times \{0\})$ and k rel $j(X_0 \times \{0\})$ from id_M to $\eta \circ \rho$ respectively $\rho \circ \eta$.