Freie Universität Berlin
Institut für Mathematik

Elmar Vogt
Filipp Levikov

Homotopy Theory

Summer 2015

Homework 6 Due: May 28, 2015

Problem 11

Let $V_{k,n} = V_{k,n}(\mathbb{R})$ be the space of orthonormal k-frames $(v_1, \ldots, v_k)$ in $\mathbb{R}^n$, i.e.

$$(v_1, \ldots, v_k) \subset \ (\mathbb{R}^n)^k \quad \text{and} \quad < v_i, v_j >= \delta_{ij} \quad \text{for all} \quad i, j = 1, \ldots, k.$$

In the lecture we have proved that

$$\pi_i(V_{k,n}) = 0, \quad \text{for} \quad 0 \leq i \leq n - k - 1.$$

Assume that for $r > 0$

$$\pi_j(S^r) = \begin{cases} 0, & 0 \leq j < n \\ \mathbb{Z}, & j = r. \end{cases}$$

and show that $\pi_{n-k}(V_{k,n})$ is cyclic.

Problem 12

Continuing the computation in Problem 11, show that for $n > k$ and $(n - k)$ even

$$\pi_i(V_{k,n}) = \mathbb{Z}.$$

Hint: Produce a section of $V_{2,n-k+2} \to S^{n-k+1}$.