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Homotopy Theory

Summer 2015

Homework 4 — Due: May 13, 2015

Note May 14 is a public holiday!

A little point set topology for cofibrations.

Problem 7

(i) In the lecture we showed that any cofibration $f : A \to X$ is injective. Basically the "same" proof will show that f is an embedding.

(ii) Show that $f(A)$ is closed if X is Hausdorff.

(iii) Let $X = \{0, 1\}$ with $X, \emptyset$ and $\{0\}$ as its open sets. Show that $\{0\} \hookrightarrow X$ is a cofibration.

Problem 8

An NDR-pair (X, A) is a topological pair (X, A) (i.e. $A \subset X$ is a subspace) which admits continuous maps $u : X \to I$ and $h : X \times I \to X$ such that

(i) $A = u^{-1}(0)$

(ii) $h(x, 0) = x, \quad x \in X$

(iii) $h(x, t) = x, \quad x \in A$

(iv) $h(x, 1) \in A, \quad x \in u^{-1}\big([0, 1)\big)$.

If (iv) holds for all $x \in X$ then (X, A) is called a DR-pair.

Show that $A \hookrightarrow X$ is a cofibration, if (X, A) is an NDR-pair.

Hint: Show that $(X \times 0 \cup A \times I) \subset X \times I$ is a DR-pair and thus a deformation retract. For $\tilde{u} : X \times I \to I$ use $\tilde{u}(x, t) = \min\{u(x), t\}$ and for $\tilde{h}$ consider the sets $t \geq u(x)$ and $t \leq u(x)$, and note that for $t > u(x)$ division by t and for $t < u(x)$ division by $u(x)$ is possible. If you cannot concoct a formula for $\tilde{h}$ consult the internet.