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Homotopy Theory

Summer 2015

Homework 10 Due: July 2, 2015

Denote by $\mathbf{Set}_*$ the category of pointed sets and by $\mathbf{Top}_*$ the category of pointed topological spaces. A contravariant functor

$$F : \mathbf{Top}_* \to \mathbf{Set}_*$$

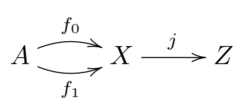
is called a *homotopy functor* if for $g_1, g_2 : X \to Y$ two homotopic maps in $\mathbf{Top}_*$, their images under F coincide: $F(g_1) = F(g_2)$. It is called a *Brown functor* if in addition the following properties are satisfied:

(i) Let $\{X_\alpha\}_{\alpha \in A}$ be a family of spaces and denote for each $\alpha \in A$ by i_α the canonical inclusions $i_\alpha : X_\alpha \hookrightarrow \bigvee_{\alpha \in A} X_\alpha$. We require that the canonical map

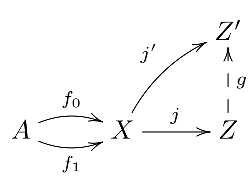
$$\{F(X_\alpha)\}_\alpha : F(\bigvee_{\alpha \in A} X_\alpha) \to \prod_{\alpha \in A} F(X_\alpha)$$

is a bijection.

(ii) Given a diagram of the form



such that $jf_1 \simeq jf_0$ and such that for every space Z' and a map j' with this property there exists a map g making the diagram commute up to homotopy



we require that: if there is a $u \in F(X)$ such that $F(f_0)(u) = F(f_1)(u)$, there is a $v \in F(Z)$ such that $F(j)(v) = u$.

(Please turn over.)

Problem 21

Let F be a Brown functor. For a based space X denote by SX the reduced suspension. Define

$$\nu : SX \to SX \vee SX, \qquad [t,x] \mapsto \begin{cases} [2t,x] \vee \{*\}, & 0 \le t \le \frac{1}{2} \\ \{*\} \vee [2t-1,x], & \frac{1}{2} \le t \le 1 \end{cases}$$

and show that the composition

$$F(SX) \times F(SX) \xrightarrow{\{F(i_1),F(i_2)\}^{-1}} F(SX \vee SX) \xrightarrow{F(\nu)} F(SX)$$

defines a group multiplication on $F(SX)$ which is abelian if X is a suspension.