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Homotopy Theory

Summer 2015

Homework 1 Due: April 23, 2015

Here are two problems to familiarize you with some of the new concepts.

Problem 1

Let G be a topological group, i. e. a group whose underlying set has a topology such that multiplication and the map $g \mapsto g^{-1}$ are continuous. Then for any space X the set of continuous maps $X \to G$ with pointwise multiplication is a group. Also the set of homotopy classes of maps $(X, A) \to (G, 1)$ is a group for any subspace A of X. (You might want to check this.) So we have two compositions on the set of homotopy classes of maps $(I^n, \partial I^n) \to (G, 1)$ if $n \geq 1$.

Show that these compositions are in fact equal and commutative. (G need not be commutative.)

Problem 2

Let $n \geq 0$ and $f : S^n \to X$ be homotopic to a constant map. Then show that there is a homotopy from f to a constant map as maps of pointed spaces $(S^n, y_0) \to (X, f(y_0))$.

You might want to think whether the above statement holds if we replace (S^n, y_0) by an arbitrary pointed space (and come up with a counterexample).