Freie Universität Berlin

Surfaces and Automorphisms

Problem Set 7 H. Reich/F. Lenhardt
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Exercise 1
Let M be a connected compact Riemann surface. Let $f\colon M \to \mathbb{P}^1$ be a nonconstant holomorphic function. Show:

(i) $S = f^{-1}(\infty)$ is finite.

(ii) For every point $z_0 \in M$ there is a $k \in \mathbb{Z}$ and a chart $\phi\colon U \to V$ with $U \subset M$ open, $z_0 \in U$, $V \subset \mathbb{C}$ with $0 \in V$ such that $\phi(z_0) = 0$ and

$$f \circ \phi^{-1}(z) = z^k \cdot h(z)$$

for some holomorphic function $h\colon V \to \mathbb{C}$ with $h(0) \neq 0$.

Exercise 2
Let M be a smooth manifold and $U, V \subset M$.

(i) Show that there is a short exact sequence of chain complexes

$$0 \to \Omega^*(U \cup V) \to \Omega^*(U) \oplus \Omega^*(V) \to \Omega^*(U \cap V) \to 0$$

(ii) Show that there is a long exact sequence in de Rham cohomology

$$\dots \to H^{n+1}_{dR}(U \cap V) \to H^n_{dR}(U) \oplus H^n_{dR}(V) \to H^n_{dR}(U \cup V) \to H^{n-1}_{dR}(U \cap V) \to \dots$$

Exercise 3
Consider $\mathbb{U} = \mathbb{P}^1, \mathbb{C}, \mathbb{H}$. Investigate whether the action of $\mathrm{Bihol}(\mathbb{U})$ on the configuration space $\mathrm{Conf}_k(\mathbb{U})$ is transitive for $k = 1, 2, 3$ and compute the stablizers of these actions.

Exercise 4
Let D be the open unit disc in $\mathbb{C}$.

(i) Find an explicit biholomorphism from D to $\mathbb{H}$.

(ii) Show that the subgroups $PSL_2(\mathbb{R}) \subset PSL_2(\mathbb{C})$ and

$$\left\{ \frac{az + b}{\bar{b}z + \bar{a}} \mid a, b \in \mathbb{C} \right\} \subset PSL_2(\mathbb{C})$$

are conjugated.