Freie Universität Berlin

Surfaces and Automorphisms

Problem Set 8 H. Reich/F. Lenhardt
WS2013/14

Exercise 1

(i) Let the cyclic groups C_n act on $\mathbb{D}$ by rotation. Show that even though $0 \in \mathbb{D}$ is a fixed point, the quotient $\mathbb{D}/C_n$ can be equipped with the structure of a Riemann surface.

(ii) Suppose $\Gamma \subset \mathrm{Bihol}(\mathbb{D})$ is a discrete subgroup. Show that for every $x \in \mathbb{D}$, the stabilizer Γ_x is trivial or finite cyclic.

(iii) Show that then $\mathbb{D}/\Gamma$ can be equipped with the structure of a Riemann surface even if Γ does not act freely.

Exercise 2

(i) Consider $A \in SL_2(\mathbb{R})$. Show that if $1 \neq [A] \in PSL_2(\mathbb{R})$ has a fixed point in $\mathbb{H}$, then $|\operatorname{tr} A| < 2$.

(ii) Let Γ_p be the kernel of the natural map $SL_2(\mathbb{Z}) \to SL_2(\mathbb{F}_p)$. Show that the image of Γ_p in $PSL_2(\mathbb{R})$ acts freely on $\mathbb{H}$ and that $\mathbb{H}/\Gamma_p$ is a Riemann surface.

Exercise 3
Let $f\colon \mathbb{D} \to \mathbb{D}$ be a holomorphic map with $f(0) = 0$.

(i) Show that $\left|\frac{\partial f}{\partial z}(0)\right| \leq 1$.

(ii) If there exists a point $w \in \mathbb{D}$ such that $|f(w)| = |w|$, then f is a rotation.

(iii) If $\left|\frac{\partial f}{\partial z}(0)\right| = 1$, then f is a rotation.

Hint: Look at the power series of f and use the maximum principle.