Configuration spaces

Problem Set 2   E. Vogt/F. Lenhardt
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Exercise 1

Let S_n be the symmetric group on n letters. Show that the configuration space $F_n(\mathbb{R})$ of n ordered particles in $\mathbb{R}$ is homotopy equivalent to S_n considered as a space with the discrete topology.

Exercise 2

Show that the configuration space $F_n(S^1)$ of n ordered particles in S^1 is homotopy equivalent to $S_{n-1} \times S^1$.

Exercise 3

The symmetric group S_n acts on $F_n(\mathbb{R})$ via $\sigma(x_1, \ldots x_n) = (x_{\sigma(1)}, \ldots x_{\sigma(n)})$. Show that the quotient $F_n(\mathbb{R})/S_n$ is contractible.

Exercise 4

Let M be a *connected* manifold and $x, y \in M$. Show that there is a homeomorphism $f \colon M \to M$ such that $f(x) = y$.
Hint: Consider for $x \in M$ fixed the set

$$\{y \in M \mid \text{there is a homeomorphism of } M \text{ sending } x \text{ to } y\}$$