Summary

Geometric packing and scaling problems are fundamental tasks in the field of *Computational Geometry*. They appear in various application areas and also in theoretical mathematics. The problems we consider in this thesis are of the following two types:
We define a *(geometric) packing problem with k degrees of freedom* as follows:

> Given a collection of *candidate sets* $C_1, C_2, \ldots, C_n$, each consisting of at most k geometric objects, where each object posseses a scaling center, find the maximal scaling factor $\delta_{\max}$ of all reals $\delta > 0$, and find $x_1, x_2, \ldots, x_n$ with $x_i \in C_i$, such that all $x_i (1 \leq i \leq n)$ scaled by δ around their scaling centers, are pairwise disjoint.

In this thesis we study geometric packing and scaling problems from an algorithmic point of view, where the candidates are two-dimensional geometric objects, like disks, squares, rectangles, and convex, orthoconvex or starshaped polygons etc.

Geometric packing problems ar $\mathcal{NP}$-hard in general, and therefore also the related optimization problems, namely the corresponding scaling problems, are in general not solvable in polynomial time, unless $\mathcal{P} = \mathcal{NP}$. We study the borderline between $\mathcal{P}$ and $\mathcal{NP}$ for these problems by limiting the degree of freedom.

Our main results lie in the area of scaling problems. For one degree of freedom we will describe a method to compute the maximal scaling factor in optimal $\mathcal{O}(n \log n)$ time for n disks with different radii. This method can be extended to a broader class of objects, including disks generated by L_p-norms ($1 \leq p \leq \infty$). Another approach, working for a wider class of objects (starshaped, x-monotone), with a runtime of $\mathcal{O}(n \log^2 n)$, where n denotes the number of objects, will also be presented.

For degree of freedom equal to two, we are able to combine the parametric search paradigm and techniques of Imai and Asano to solve the scaling problem efficiently for example for rectangles, starshaped orthoconvex objects, and for disks.

In the case of four degrees of freedom we will show, that for the following special case we can efficiently compute in $\mathcal{O}(n \log n)$ time a 50% approximation of the maximal scaling factor: For a collection of n distinct points (sites), let each one have as candidate set the four different unit-squares with the site in one of its corners, each with its site as its scaling center.

We remark that the approximation quality of 50% is the best possbile achievable in polynomial time, unless $\mathcal{P} = \mathcal{NP}$. We will generalize the technique to other object shapes. Finally, we will show, that also a special 3-dimensional scaling problem with 6 degrees of freedom can be solved approximately, using our technique. In this case, the approximation algorithm is guaranteed to achieve at lease $33\frac{1}{3}\%$ of the optimal solution.