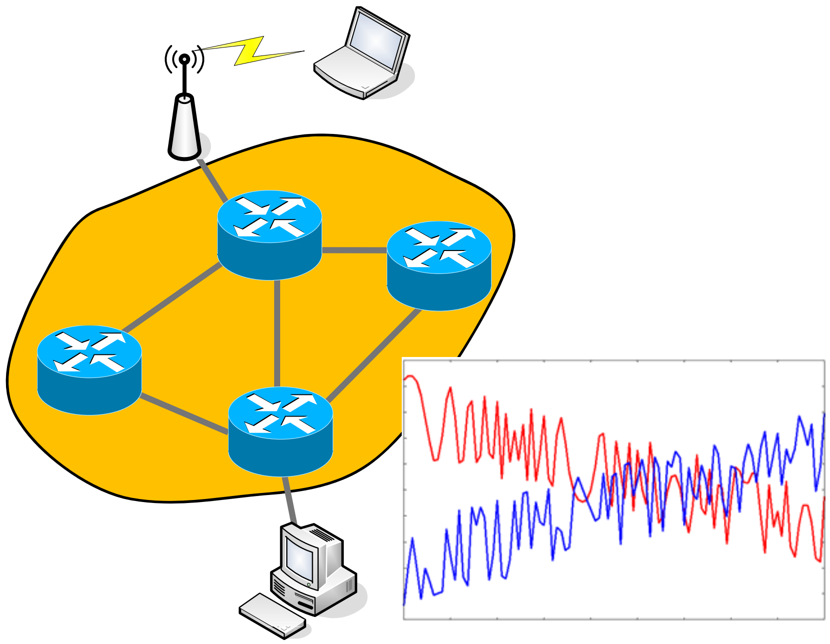
Chapter 10

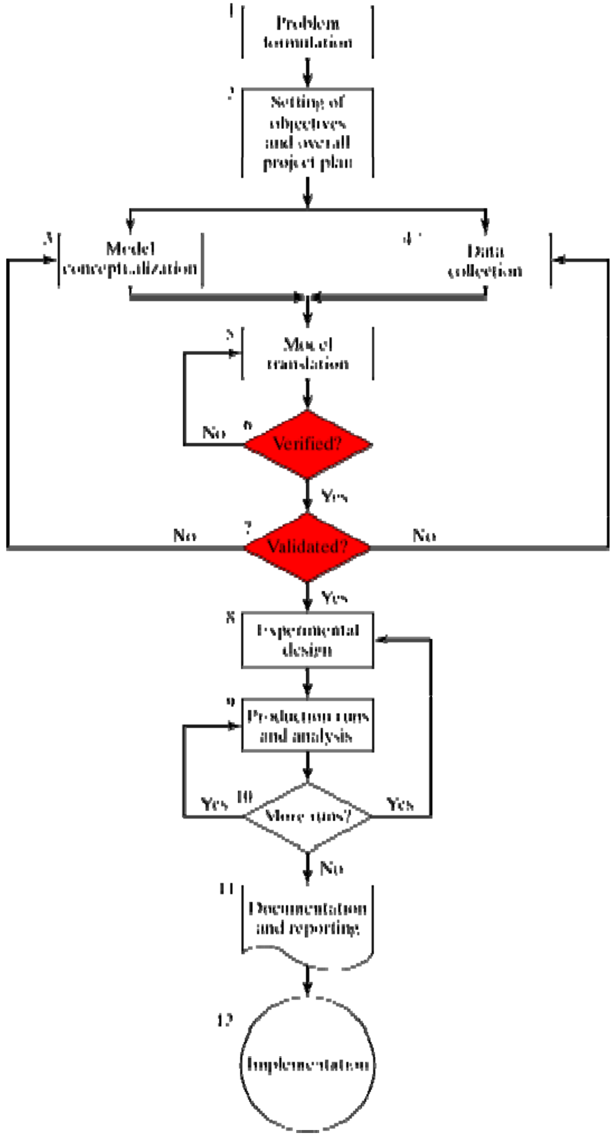
Verification and Validation of Simulation Models



Contents

Purpose & Overview

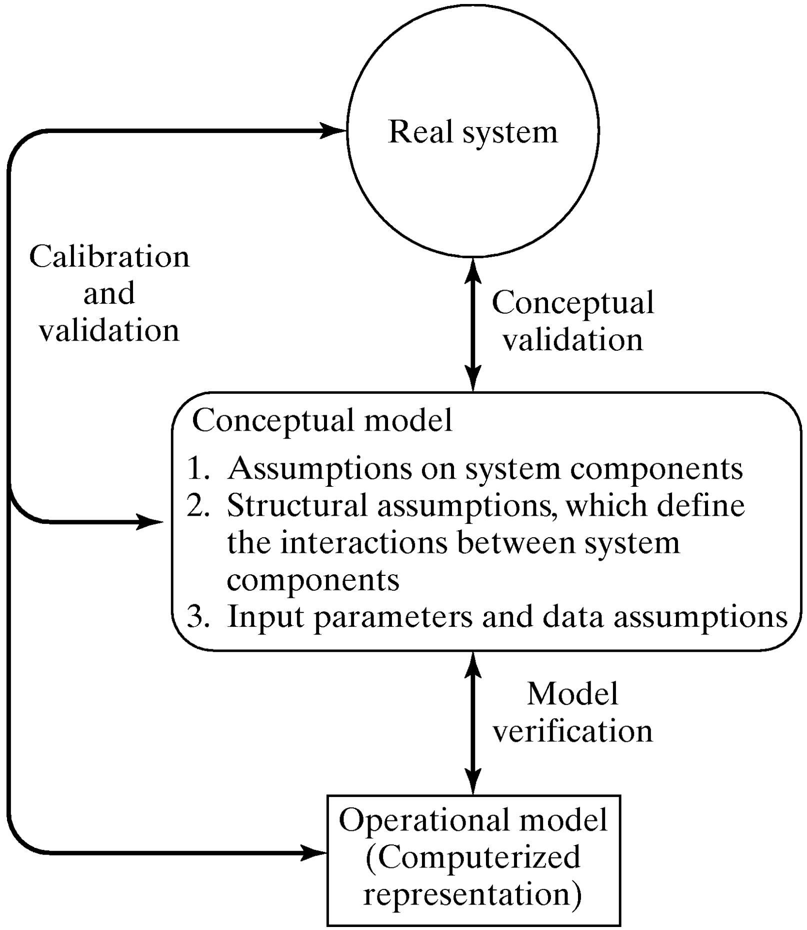
- The goal of the validation process is:
  - To produce a model that represents true behavior closely enough for decision-making purposes
  - To increase the model's credibility to an acceptable level
- Validation is an integral part of model development:
  - **Verification:** building the model correctly, correctly implemented with good input and structure
  - **Validation:** building the correct model, an accurate representation of the real system
- Most methods are informal subjective comparisons while a few are formal statistical procedures



Modeling-Building, Verification & Validation

Modeling-Building, Verification & Validation

- Steps in Model-Building
  - Observing the real system and the interactions among their various components and of collecting data on their behavior
  - Construction of a conceptual model
  - Implementation of an operational model



Verification

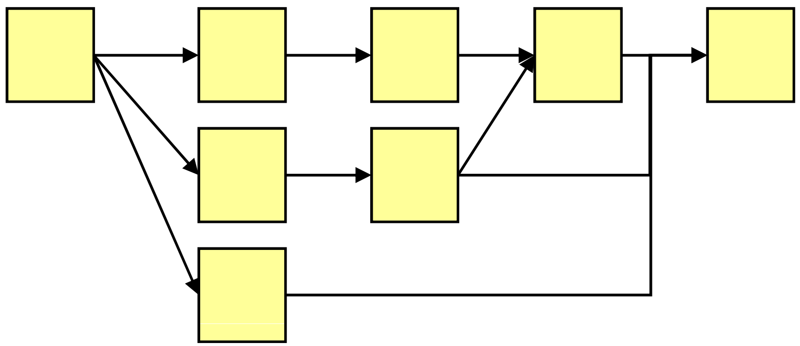
- Purpose: ensure the conceptual model is reflected accurately in the computerized representation.
- Many common-sense suggestions, for example:
  - Have someone else check the model.
  - Make a flow diagram that includes each logically possible action a system can take when an event occurs.
  - Closely examine the model output for reasonableness under a variety of input parameter settings.
  - Print the input parameters at the end of the simulation, make sure they have not been changed inadvertently.
  - Make the operational model as self-documenting as possible.
  - If the operational model is animated, verify that what is seen in the animation imitates the actual system.
  - Use the debugger.
  - If possible use a graphical representation of the model.

Examination of Model Output for Reasonableness

- Two statistics that give a quick indication of model reasonableness are current contents and total counts
  - Current content: The number of items in each component of the system at a given time.
  - Total counts: Total number of items that have entered each component of the system by a given time.
- Compute certain long-run measures of performance, e.g. compute the long-run server utilization and compare to simulation results.

Examination of Model Output for Reasonableness

- A model of a complex network of queues consisting of many service centers.
  - If the current content grows in a more or less linear fashion as the simulation run time increases, it is likely that a queue is unstable
  - If the total count for some subsystem is zero, indicates no items entered that subsystem, a highly suspect occurrence
  - If the total and current count are equal to one, can indicate that an entity has captured a resource but never freed that resource.



Other Important Tools

- Documentation
  - A means of clarifying the logic of a model and verifying its completeness.
  - Comment the operational model, definition of all variables and parameters.
- Use of a trace
  - A detailed printout of the state of the simulation model over time.
    - Can be very labor intensive if the programming language does not support statistic collection.
    - Labor can be reduced by a centralized tracing mechanism
    - In object-oriented simulation framework, trace support can be integrated into class hierarchy. New classes need only to add little for the trace support.

Trace: Example

- Simple queue from Chapter 2
- Trace over a time interval [0, 16]
- Allows the test of the results by pen-and-paper method

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Definition of Variables:
CLOCK = Simulation clock
EVTYP = Event type (Start, Arrival, Departure, Stop)
NCUST = Number of customers in system at time CLOCK
STATUS = Status of server (1=busy, 0=idle)

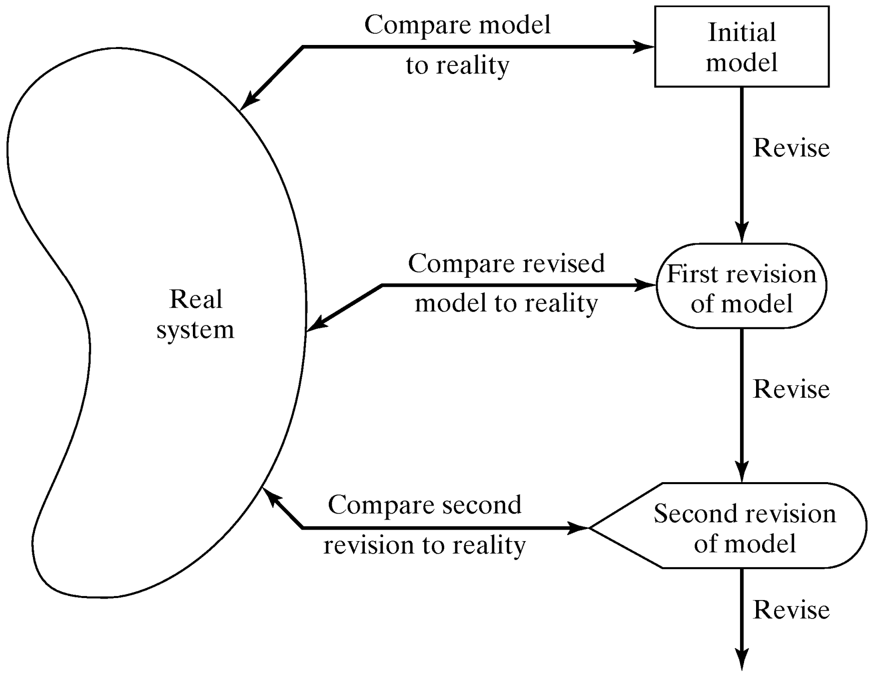
State of System Just After the Named Event Occurs:
CLOCK =  0  EVTYP = Start    NCUST=0  STATUS = 0
CLOCK =  3  EVTYP = Arrival  NCUST=1  STATUS = 0
CLOCK =  5  EVTYP = Depart   NCUST=0  STATUS = 0
CLOCK = 11  EVTYP = Arrival  NCUST=1  STATUS = 0
CLOCK = 12  EVTYP = Arrival  NCUST=2  STATUS = 1
CLOCK = 16  EVTYP = Depart   NCUST=1  STATUS = 1
...
```

There is a customer, but the status is 0

Calibration and Validation

Calibration and Validation

- Validation: the overall process of comparing the model and its behavior to the real system.
- Calibration: the iterative process of comparing the model to the real system and making adjustments.
- Comparison of the model to real system
  - Subjective tests
    - People who are knowledgeable about the system
  - Objective tests
    - Requires data on the real system's behavior and the output of the model



Calibration and Validation

- Danger during the calibration phase
  - Typically few data sets are available, in the worst case only one, and the model is only validated for these.
  - Solution: If possible collect new data sets

- No model is ever a perfect representation of the system
  - The modeler must weigh the possible, but not guaranteed, increase in model accuracy versus the cost of increased validation effort.

- Three-step approach for validation:
  - Build a model that has high face validity.
  - Validate model assumptions.
  - Compare the model input-output transformations with the real system's data.

High Face Validity

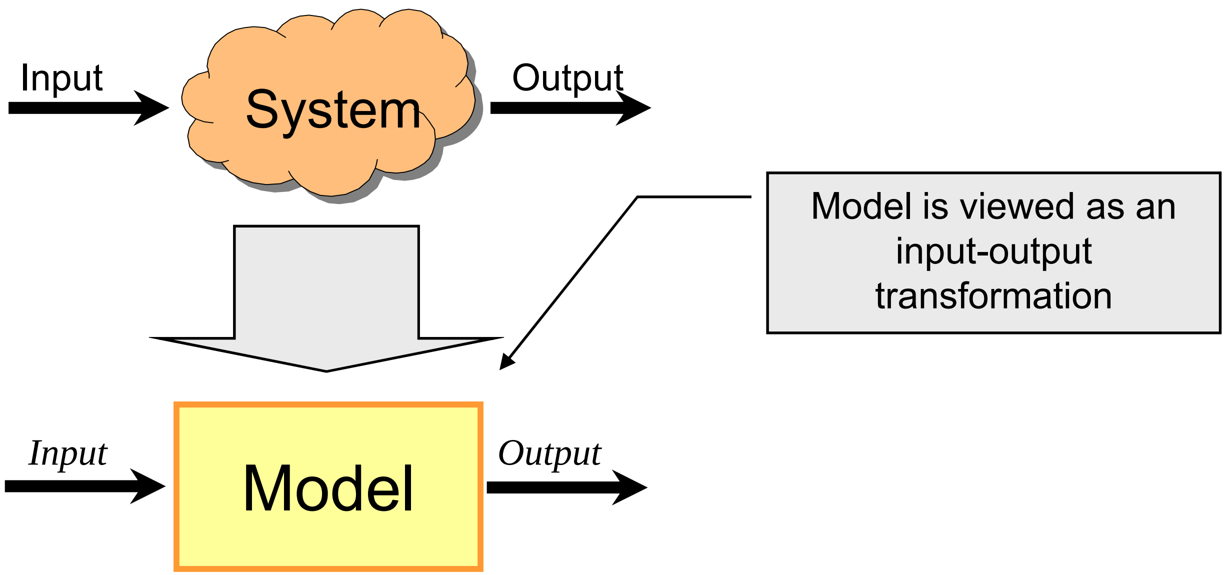
- Ensure a high degree of realism:
  - Potential users should be involved in model construction from its conceptualization to its implementation.
- Sensitivity analysis can also be used to check a model's face validity.
  - Example: In most queueing systems, if the arrival rate of customers were to increase, it would be expected that server utilization, queue length and delays would tend to increase.
  - For large-scale simulation models, there are many input variables and thus possibly many sensitivity tests.
    - Sometimes not possible to perform all of theses tests, select the most critical ones.

Validate Model Assumptions

- General classes of model assumptions:
  - Structural assumptions: how the system operates.
  - Data assumptions: reliability of data and its statistical analysis.
- Bank example: customer queueing and service facility in a bank.
  - Structural assumptions
    - Customer waiting in one line versus many lines
    - Customers are served according FCFS versus priority
  - Data assumptions, e.g., interarrival time of customers, service times for commercial accounts.
    - Verify data reliability with bank managers
    - Test correlation and goodness of fit for data

Validate Input-Output Transformation

- Goal: Validate the model's ability to predict future behavior
  - The only objective test of the model.
  - The structure of the model should be accurate enough to make good predictions for the range of input data sets of interest.
- One possible approach: use historical data that have been reserved for validation purposes only.
- Criteria: use the main responses of interest.



Bank Example

- Example: One drive-in window serviced by one teller, only one or two transactions are allowed.
  - Data collection: 90 customers during 11 am to 1 pm.
    - Observed service times $\{S_i, i = 1,2, \ldots, 90\}$.
    - Observed interarrival times $\{A_i, i = 1,2, \ldots, 90\}$.
  - Data analysis let to the conclusion that:
    - Interarrival times: exponentially distributed with rate λ = 45/hour
    - Service times: $N(1.1, 0.2^2)$

Input variables

Bank Example: The Black Box

- A model was developed in close consultation with bank management and employees
- Model assumptions were validated
- Resulting model is now viewed as a "black box":

Uncontrolled variables, X

Controlled Decision variables, D

Input Variables

Poisson arrivals
$\lambda = 45/hr: X_{11}, X_{12}, \ldots$
Services times,
$N(D_2, 0.22): X_{21}, X_{22}, \ldots$

$D_1 = 1$ (one teller)
$D_2 = 1.1$ min
(mean service time)
$D_3 = 1$ (one line)

Model "black box"
$f(X,D) = Y$

Model Output Variables, Y

Primary interest:
Y_1 = teller's utilization
Y_2 = average delay
Y_3 = maximum line length

Secondary interest:
Y_4 = observed arrival rate
Y_5 = average service time
Y_6 = sample std. dev. of service times
Y_7 = average length of time

Comparison with Real System Data

- Real system data are necessary for validation.
  - System responses should have been collected during the same time period (from 11am to 1pm on the same day.)
- Compare average delay from the model Y_2 with actual delay Z_2:
  - Average delay observed, $Z_2 = 4.3$ minutes, consider this to be the true mean value $\mu_0 = 4.3$.
  - When the model is run with generated random variates X_{1n} and X_{2n}, Y_2 should be close to Z_2.

Comparison with Real System Data

- Six statistically independent replications of the model, each of 2-hour duration, are run.

Replication	Y_4 Arrivals/Hour	Y_5 Service Time [Minutes]	Y_2 Average Delay [Minutes]
1	51.0	1.07	2.79
2	40.0	1.12	1.12
3	45.5	1.06	2.24
4	50.5	1.10	3.45
5	53.0	1.09	3.13
6	49.0	1.07	2.38
Sample mean [Delay]			**2.51**
Standard deviation [Delay]			**0.82**

Hypothesis Testing

- Compare the average delay from the model Y_2 with the actual delay Z_2

  - Null hypothesis testing: evaluate whether the simulation and the real system are the same (w.r.t. output measures):

$$H_0\text{: } E(Y_2) = 4.3 \text{ minutes}$$

$$H_1\text{: } E(Y_2) \neq 4.3 \text{ minutes}$$

    - If H_0 is not rejected, then, there is no reason to consider the model invalid
    - If H_0 is rejected, the current version of the model is rejected, and the modeler needs to improve the model

Hypothesis Testing

- Conduct the t test:
  - Chose level of significance ($\alpha = 0.05$) and sample size ($n = 6$).
  - Compute the sample mean and sample standard deviation over the n replications:

$$\bar{Y}_2 = \frac{1}{n}\sum_{i=1}^{n} Y_{2i} = 2.51 \text{ minutes} \qquad S = \sqrt{\frac{\sum_{i=1}^{n}(Y_{2i} - \bar{Y}_2)^2}{n-1}} = 0.82 \text{ minutes}$$

  - Compute test statistics:

$$|t_0| = \left|\frac{\bar{Y}_2 - \mu_0}{S/\sqrt{n}}\right| = \left|\frac{2.51 - 4.3}{0.82/\sqrt{6}}\right| = 5.34 \;>\; t_{critical} = 2.571 \quad \text{(for a 2-sided test)}$$

  - Hence, reject H_0.
    - Conclude that the model is inadequate.
  - Check: the assumptions justifying a t test, that the observations (Y_{2i}) are normally and independently distributed.

Hypothesis Testing

- Similarly, compare the model output with the observed output for other measures:

$$Y_4 \leftrightarrow Z_4,\ Y_5 \leftrightarrow Z_5,\ and\ Y_6 \leftrightarrow Z_6$$

Type II Error

- For validation, the power of the test is:
  - Probability(detecting an invalid model) = 1 – β
  - $\beta = \text{P(Type II error)} = \text{P(failing to reject } H_0 \mid H_1 \text{ is true)}$
  - Consider failure to reject H_0 as a strong conclusion, the modeler would want β to be small.
  - Value of β depends on:
    - Sample size, n
    - The true difference, δ, between $E(Y)$ and μ:

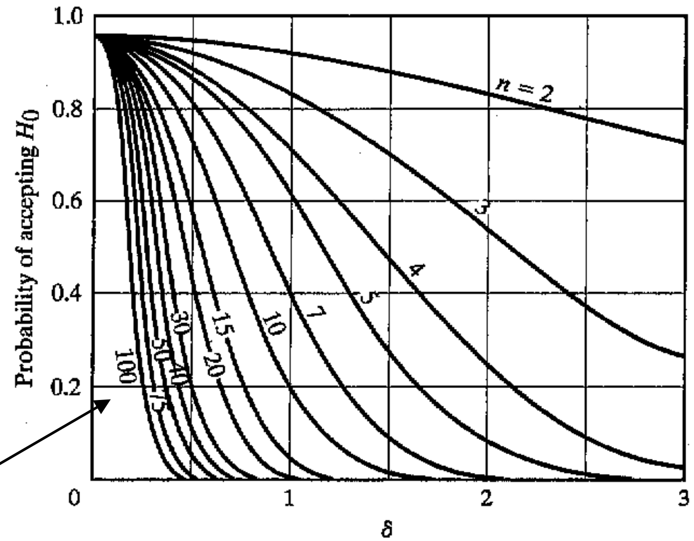
$$\delta = \frac{|E(Y) - \mu|}{\sigma}$$

- In general, the best approach to control β error is:
  - Specify the critical difference, δ.
  - Choose a sample size, n, by making use of the operating characteristics curve (OC curve).

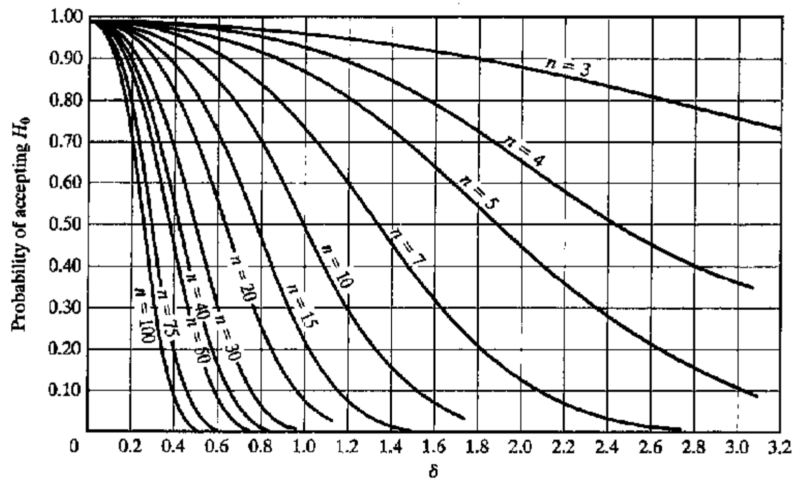
Type II Error

- Operating characteristics curve (OC curve).
  - Graphs of the probability of a Type II Error $\beta(\delta)$ versus δ for a given sample size n

For the same error probability with smaller difference the required sample size increases!



(a) $\alpha = 0.05$



(b) $\alpha = 0.01$

Type I and II Error

- Type I error (α):
  - Error of rejecting a valid model.
  - Controlled by specifying a small level of significance α.
- Type II error (β):
  - Error of accepting a model as valid when it is invalid.
  - Controlled by specifying critical difference and find the n.
- For a fixed sample size n, increasing α will decrease β.

Statistical Terminology	Modeling Terminology	Associated Risk
Type I: rejecting H_0 when H_0 is true	**Rejecting a valid model**	α
Type II: failure to reject H_0 when H_1 is true	**Failure to reject an invalid model**	β

Confidence Interval Testing

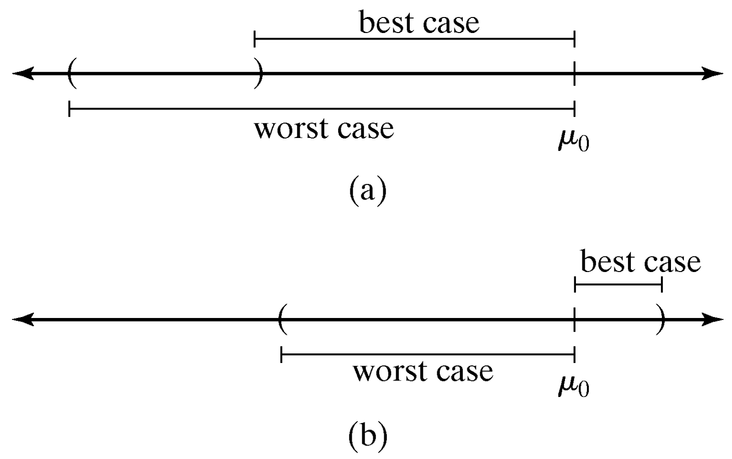
- Confidence interval testing: evaluate whether the simulation and the real system performance measures are close enough.
- If Y is the simulation output, and $\mu = E(Y)$
- The confidence interval (CI) for μ is:

$$\left[\bar{Y} - t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}, \bar{Y} + t_{\frac{\alpha}{2},n-1}\frac{S}{\sqrt{n}}\right]$$

Confidence Interval Testing

- Validating the model:
  - Suppose the CI does not contain μ_0:
    - If the best-case error is > ε, model needs to be refined.
    - If the worst-case error is ≤ ε, accept the model.
    - If best-case error is ≤ ε, additional replications are necessary.
  - Suppose the CI contains μ_0:
    - If either the best-case or worst-case error is > ε, additional replications are necessary.
    - If the worst-case error is ≤ ε, accept the model.

ε is a difference value chosen by the analyst, that is small enough to allow valid decisions to be based on simulations!



Confidence Interval Testing

- Bank example: $\mu_0 = 4.3$, and "close enough" is ε = 1 minute of expected customer delay.
  - A 95% confidence interval, based on the 6 replications is [1.65, 3.37] because:

$$\bar{Y} \pm t_{0.025,5} \frac{S}{\sqrt{n}}$$

$$2.51 \pm 2.571 \frac{0.82}{\sqrt{6}}$$

  - $\mu_0 = 4.3$ falls outside the confidence interval,
    - the best case |3.37 – 4.3| = 0.93 < 1, but
    - the worst case |1.65 – 4.3| = 2.65 > 1
    - ➨ Additional replications are needed to reach a decision.

Using Historical Output Data

- An alternative to generating input data:
  - Use the actual historical record.
  - Drive the simulation model with the historical record and then compare model output to system data.
  - In the bank example, use the recorded interarrival and service times for the customers $\{A_n, S_n, n = 1,2,\ldots\}$.

- Procedure and validation process: similar to the approach used for system generated input data.

Using a Turing Test

- Use in addition to statistical test, or when no statistical test is readily applicable.

> **Turing Test**
> Described by Alan Turing in 1950. A human jugde is involved in a natural language conversation with a human and a machine. If the judge cannot reliably tell which of the partners is the machine, then the machine has passed the test.

- Utilize persons' knowledge about the system.
- For example:
  - Present 10 system performance reports to a manager of the system. Five of them are from the real system and the rest are "fake" reports based on simulation output data.
  - If the person identifies a substantial number of the fake reports, interview the person to get information for model improvement.
  - If the person cannot distinguish between fake and real reports with consistency, conclude that the test gives no evidence of model inadequacy.

Summary

- Model validation is essential:
  - Model verification
  - Calibration and validation
  - Conceptual validation
- Best to compare system data to model data, and make comparison using a wide variety of techniques.
- Some techniques that we covered:
  - Insure high face validity by consulting knowledgeable persons.
  - Conduct simple statistical tests on assumed distributional forms.
  - Conduct a Turing test.
  - Compare model output to system output by statistical tests.