Formalization and Assessment of Lowe's Modal Ontological Argument

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The following variant of the *ontological argument* for the existence of God has been put forward by E. J. Lowe [4]:

- P1 God is, by definition, a necessary concrete being.
- P2 Some necessary abstract beings exist.
- P3 All abstract beings are dependent beings.
- P4 All dependent beings depend for their existence on independent beings.
- P5 No contingent being can explain the existence of a necessary being.
- P6 The existence of any dependent being needs to be explained.
- P7 Dependent beings of any kind cannot explain their own existence.
- P8 The existence of dependent beings can only be explained by beings on which they depend for their existence.
- C A necessary concrete being exists.

Using two different formalisation alternatives we have assessed this argument in the interactive theorem prover Isabelle/HOL.

The first alternative is modeled in *quantified modal logic* (QML) and it utilizes a actualist version of the semantical embedding of QML in classical higher-order logic (HOL) [2]. To achieve actualist (first-order) quantifiers an explicit existence predicate is introduced in the HOL meta-language and used to appropriately guard the standard possibilist quantifiers; this well known technique has been successfully applied in practice before [3].

Our experiments in Isabelle/HOL confirm that the conclusion already follows from P2: $\exists x.\Box(\mathit{Abstract}\ x)$, P3: $\forall x.\mathit{Abstract}\ x \rightarrow \mathit{Dependent}\ x$, and P4: $\forall x.\mathit{Dependent}\ x \rightarrow (\exists y.\mathit{Independent}\ y \wedge x\ \mathit{dependsOn}\ y)$, when we additionally assume that concreteness is a necessary property of beings: $\forall x.\mathit{Concrete}\ x \rightarrow \Box(\mathit{Concrete}\ x)$.[1] In our modeling, *Concrete* is an uninterpreted rigid predicate symbol and *Abstract* x is an abbreviation for $\neg(\mathit{Concrete}\ x)$. Moreover, *Dependend* x and *Independend* x are abbreviations for $\exists y.x\ \mathit{dependsOn}\ y$ and

[1]EDNOTE: David please say what happens if we omit this additional assumption?

$\neg(Dependend\ x)$ respectively, where *dependsOn* is an uninterpreted rigid relation symbol. Assuming a global semantics for the logical consequence relation Isabelle/HOL then proves that there are necessarily concrete objects (in every world) C: $\exists x.\Box(Concrete\ x)$.

The second alternative is a non-modal formalisation in pure *first-order predicate logic*. It is motivated by a simplified, literal reading of the premises and the conclusion, while the above formalisation presumably better honors the original intention of Lowe. According to Lowe *"there is no logical restriction on combinations of the properties involved in the* concrete/abstract *and the* necessary/contingent *distinctions. Thus, in principle, we can have contingent concrete beings, contingent abstract beings, necessary concrete beings, and necessary abstract beings."* By taking these four categories as exhausting our domain of discourse, a different reading of *necessity* and *contingency* reveals itself, not as modals, but as mutually exclusive predicates. As a consequence, our universe of discourse (and some exemplary members) would look as follows:

	Abstract	Concrete
Necessary	*Numbers*	*God*
Contingent	*Fiction*	*Stuff*

Our experiments in Isabelle/HOL confirm that the conclusion C: $\exists x.Necessary\ x \wedge Concrete\ x$ follows from premises P2: $\exists x.Necessary\ x \wedge Abstract\ x$, P3: $\forall x.Abstract\ x \rightarrow Dependent\ x$, P4: $\forall x.Dependent\ x \rightarrow (\exists y.Independent\ y \wedge x\ dependsOn\ y)$ and P5: $\forall x.Necessary\ x \rightarrow (\forall y.x\ dependsOn\ y \rightarrow Necessary\ y)$. Here, $Necessary$ and $Concrete$ are uninterpreted constant symbols and $Contingent\ x$ and $Abstract\ x$ are abbreviations for $\neg(Necessary\ x)$ and $\neg(Concrete\ x)$, respectively. *Dependent*, *Independent* and *dependsOn* are modeled analogous to before.

The ambiguity of natural language for different formalizations of the same argument, two of which we have formalised in Isabelle/HOL as outlined above. The first variant tries to capture the essentialist nature of the concreteness predicate, and the second exploits the very idiosyncratic meaning given by the author to the terms necessity and contingency inside his argument. The full details of our formalisations and experiments are available online[2]. Note that in both of our formalisations only a subset of Lowe's premises is needed to justify the conclusion. Moreover, in both variants the consistency of the premises was confirmed. We invite the readers to inspect and adapt our formalisations, and to eventually contribute further alternative formal interpretations of Lowe's natural language argument.

The work presented here is a result of a student project of the (awarded) lecture course on *Computational Metaphysics* at held in Summer 2016 at FU Berlin. In this lecture course we pioneered the rigorous, deep logical assessment of rational arguments in philosophy on the computer; for more details see [1].

[2] See `http://christoph-benzmueller.de/papers/2017-Lowe-OntologicalArgument.zip`

References

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