

# Berlin-Hamburg-Poznań Seminar

08-09 June 2018

Freie Universität Berlin

## Programm

Friday, June 8<sup>th</sup>, 2018

13:30 – 14:00 *Arrival of Participants*

14:00-14:45 Maciej Dołęga (Poznań)

### Enumeration of maps and Jack polynomials

Enumerative combinatorics is a classical part of combinatorics where one tries to find the number of objects with a given set of properties. The objects I am going to enumerate in my talk are maps, which are graphs embedded into surfaces. I will discuss a remarkable connection of maps enumeration with symmetric functions and algebraic combinatorics. In particular, I am going to focus on the b-Conjecture posed by Goulden and Jackson in 1996 which relates Jack symmetric functions with maps enumeration and I explain the recent progress in this area.

14:45-15:15 Wiebke Bedenknecht (Hamburg)

### Squares of Hamiltonian cycles in 3-uniform hypergraphs

We show that every 3-uniform hypergraph  $H=(V,E)$  with  $|V(H)|=n$  and minimum pair degree at least  $(4/5+o(1))n$  contains a squared Hamiltonian cycle. This may be regarded as a first step towards a hypergraph version of the Pósa-Seymour conjecture.

15:15 – 15:45 *Coffee break*

17:15 – 17:30 *Coffee break*

17:30-18:00 Joanna Polcyn (Poznań)

### A tale of stars and cliques

We show that for infinitely many natural numbers  $k$  there are  $k$ -uniform hypergraphs which admit a 'rescaling phenomenon'. More precisely, let  $A(k, l, n)$  denote the class of  $k$ -graphs on  $n$  vertices in which the sizes of all pairwise intersections of edges belong to a set  $l$ . We show that if  $k = rt^2$  for some  $r \geq 1$  and  $t \geq 2$ , and  $l$  is chosen in some special way, the densest graphs in  $A(rt^2, l, n)$  are either dominated by stars of large degree, or basically, they are ' $t$ -thick'  $rt^2$ -graphs in which vertices are partitioned into groups of  $t$  vertices each and every edge is a union of  $tr$  such groups. It is easy to see that, unlike in stars, the maximum degree of  $t$ -thick graphs is of a lower order than the number of its edges.

Thus, if we study the graphs from  $A(rt^2, l, n)$  with a prescribed number of edges  $m$  which minimize the maximum degree, around the value of  $m$  which is the number of edges of the largest  $t$ -thick graph, a rapid, discontinuous phase transition can be observed. Interestingly, these two types of  $k$ -graphs determine the structure of all hypergraphs in  $A(rt^2, l, n)$ . Namely, we show that each such hypergraph can be decomposed into a  $t$ -thick graph  $H_T$ , a special collection  $H_S$  of stars, and a sparse 'left-over' graph  $H_R$ .

Joint work with Tomasz Łuczak and Christian Reiher

18:00-18:30: Oliver Ebsen (Hamburg)

### Homomorphism threshold for graphs

The interplay of minimum degree and 'structural properties' of large graphs with a given forbidden subgraph is a central topic in extremal graph theory. For a given graph  $F$  we define the homomorphism threshold as the infimum  $\alpha$  such that every  $n$ -vertex  $F$ -free graph  $G$  with minimum degree  $\alpha n$  has a homomorphic image  $H$  of bounded size (independent of  $n$ ), which is  $F$ -free as well. Without the restriction of  $H$

15:45-16:15 Ander Lamaison (Berlin)

### Ramsey Density of Infinite Paths

In a two-colouring of the edges of the complete graph on the natural numbers, what is the densest monochromatic infinite path that we can always find? We measure the density of a path by the upper asymptotic density of its vertex set. This question was first studied by Erdős and Galvin, who proved that the best density is between  $2/3$  and  $8/9$ . We settle this question by proving that we can always find a monochromatic path of upper density at least  $(12 + \sqrt{8})/17 = 0.87226\dots$ , and constructing a two-colouring in which no denser path exists. This represents joint work with Jan Corsten, Louis DeBiasio and Richard Lang.

16:15-16:45 Paweł Rzażewski (Warsaw)

### Finding graph homomorphisms in subexponential time: string graphs and $P_t$ -free graphs

We consider the problem of determining the existence of a homomorphism from a graph  $G$  to a fixed graph  $H$ , respecting some additional constraints, e.g. lists or weights. If  $G$  is a string graph or a  $P_t$ -free graph, we fully characterize graphs  $H$ , for which the problem can be solved in subexponential time (i.e.,  $2^{o(n)}$ , where  $n$  is the number of vertices).

The results are obtained with Karolina Okrasa (Warsaw University of Technology).

16:45-17:15 Dennis Clemens (Hamburg)

### On the size Ramsey number of the grid

Given two graphs  $G$  and  $H$ , we say that  $G$  is a Ramsey graph for  $H$  if in every 2-colouring of the edges of  $G$  there is a monochromatic copy of  $H$ . The size Ramsey number  $r(H)$  then is defined as the minimum edge number over all graphs  $G$  being Ramsey for  $H$ . We prove that, if  $H$  is the grid graph on  $n \times n$  vertices, its size Ramsey number is bounded from above by  $n^{3+o(1)}$ . Joint work with Meysam Miralaei, Damian Reding, Mathias Schacht and Anusch Taraz.

being  $F$ -free we recover the definition of the chromatic threshold, which was determined for every graph  $F$  by Allen et al. The homomorphism threshold is less understood and we present recent joint work with M. Schacht on the homomorphism threshold for odd cycles.

18:30-19:00: Jarek Grytczuk (Warsaw)

### Some remarks on list colorings and graph polynomials

Graph polynomial  $P$  of a given graph  $G$  is a multivariable polynomial defined so that points at which it is not vanishing are precisely proper colorings of  $G$ . By studying algebraic properties of  $P$  one may derive useful information on graph colorings. For instance, the celebrated Combinatorial Nullstellensatz of Alon gives a sufficient condition for existence of a coloring from arbitrary lists of appropriate size. I will present some problems and results showing that this approach to graph coloring is perhaps not fully explored.

19:30 *Dinner*

Saturday, June 9<sup>th</sup>, 2018

9:30-10:00: Max Pitz (Hamburg)

### Updates on the Ubiquity conjecture

A graph  $G$  is said to be ubiquitous if every host graph containing arbitrarily many disjoint copies of  $G$  must contain infinitely many disjoint copies of  $G$  (where the meaning of 'copy' could be as subgraphs or as minors). For finite graphs  $G$ , ubiquity follows by greedily picking further copies. But one of the most important problems in the theory of infinite graphs is the so-called Ubiquity Conjecture due to Thomas Andreae, namely that every locally finite connected graph is ubiquitous with respect to minors.

In a series of papers in preparation, we have recently made some progress towards the ubiquity conjecture. In this talk, I will give an overview of the conjecture, explain why a greedy process cannot work,

what ends of graphs have to do with the problem, and briefly discuss some of our new results.

This is joint with N. Bowler, C. Elbracht, J. Erde, P. Gollin, K. Heuer and M. Teegen (all U. Hamburg).

10:00-10:30: Anurag Bishnoi (Berlin)

### Improved bounds on the cage number

The cage problem asks for the smallest number  $c(k, g)$  of vertices in a  $k$ -regular graph of girth  $g$ . The  $(k, g)$ -graphs which have  $c(k, g)$  vertices are known as cages. Cages are known to exist for all integers  $k \geq 2$  and  $g \geq 3$ , but an explicit construction is known only for some small values of  $k$ ,  $g$  and three infinite families where  $g \in \{6, 8, 12\}$  and  $k - 1$  is a prime power: corresponding to the generalized  $g/2$ -gons of order  $k - 1$ . When  $k - 1$  is not a prime power and  $g \in \{6, 8, 12\}$  we can construct small  $(k, g)$  graphs by picking a prime power  $q \geq k$  and then looking at the smallest induced  $k$ -regular subgraphs of the incidence graph of a generalized  $g/2$ -gon of order  $q$ . This approach has been used to obtain some of the best known upper bounds on  $c(k, g)$  for these values of  $g$ .

In this talk I will present new constructions in generalized quadrangles and hexagons which improve the known upper bounds on  $c(q, 8)$  and  $c(q, 12)$  when  $q$  is a prime power (and  $q - 1$  is not a prime power). Moreover, we will see a lower bound on the number of vertices in a regular induced subgraph of a generalized polygon, which gives us a limit on the best that we can do by this approach [1].

[1] J. Bamberg, A. Bishnoi and G. Royle. "On regular induced subgraphs of generalized polygons", J. Combin. Theory Ser. A, to appear, 2018.

10:30 – 11:00 Coffee break

11:00-11:30: Kasia Rybarczyk (Poznań)

### GHS algorithm on a graph with random weights

In many distributed systems, such as for example internet network, ad hoc networks, or sensor networks, there are many entities which operate in the system. The entities are active at any moment, may perform some local computations and have ability to communicate with each other to achieve some common goals. Algorithms working in such systems are called distributed.

We will ask about the performance of the classical distributed algorithms in the average case. We will consider a complete graph with independent random weights on edges. In this random weighted graph we will analyze the performance of a classical distributed algorithm constructing minimal spanning tree, called GHS algorithm. We will present some partial results and concentrate on interesting open questions which appear naturally in the context.

11:30-12:00: Damian Reding (Hamburg)

### Multicolour Ramsey equivalence

Two graphs are  $q$ -Ramsey equivalent if they share the same minimal  $q$ -colour Ramsey graphs. Answering questions of Szabo et al., Fox et al. proved that  $K_n$  is not 2-equivalent to  $K_n \cdot K_2$  while Bloom & Liebenau proved that it is 2-equivalent to  $K_n + K_{\{n-1\}}$ . We extend both results to an arbitrary number of colours and also, by utilizing the methods of multicolour signal senders and indicators, establish some general non-equivalence results for a pair of 3-connected graphs.

12:00-12:45: Shagnik Das (Berlin)

### Colourings without monochromatic chains

In 1974, Erdős and Rothschild introduced a colourful extremal problem, asking which  $n$ -vertex graph has the maximum number of monochromatic-triangle-free red/blue edge-colourings. While this original problem strengthens Mantel's theorem, recent years have witnessed the study of the Erdős-Rothschild extension of several classic combinatorial theorems. In this talk, we seek the Erdős-Rothschild extension of Sperner's Theorem. More precisely, we search for the set families in  $2^{[n]}$  with the most monochromatic- $k$ -chain-free  $r$ -colourings. Time and interest permitting, we shall present some results, sketch some proofs, and offer many open problems.

This is joint work with Roman Glebov, Benny Sudakov and Tuan Tran.

**Location:**  
Freie Universität Berlin  
Institut für Mathematik  
Lecture Room 001  
Animallee 3  
14195 Berlin

**Internet:**  
1. Eduroam  
2. Local network:  
Login = "conference", Password = to be given at the seminar

**Friday's Dinner:**  
Restaurant Luise  
Königin-Luise-Str. 40 – 42  
14195 Berlin

