Berlin-Poznań Seminar on Discrete Mathematics

9-10 November 2012
Konrad-Zuse-Zentrum für Informationstechnik Berlin

The Berlin-Poznań Seminar is a joint seminar organized by the three Berlin Universities (FU, HU and TU Berlin) and the Adam Mickiewicz University in Poznań. The topics include discrete mathematics and algorthims, enumerative, extremal and probabilistic combinatorics. It is supported by DFG within the Research Training Group (Graduiertenkolleg) "Methods for Discrete Structures" and the Berlin Mathematical School and by the Adam Mickiewicz University in Poznań.

Program

Friday

14:15-14:20 Opening remarks
14:20-15:20 Raman Sanyal, *Order preserving maps, polytopes, and applications*
15:20-16:00 Coffee break
16:00-16:30 Krzysztof Krzywdziński, *A new approach to generalise the Ramsey numbers*
16:30-17:00 Sylwia Antoniuk, *Collapse of random group*
17:00-17:30 Kolja Knauer, *Topological representation of planar partial cubes*
19:00- Dinner at Alter Krug

Saturday

9:30-10:30 Małgosia Bednarska-Bzdęga, *Degree and Small-graph Avoider-Enforcer games*
10:30-11:15 Coffee break
11:15-11:45 Anita Liebenau, *On the largest tournament Maker can build*
11:45-12:15 Kasia Rybarczyk-Krzywdzińska, *The coupling method for inhomogenious random intersection graphs*
12:15-12:45 Jan-Philipp Kappmeier, *Abstract flows over time*
12:45-13:00 Closing remarks and group picture

Location

Konrad-Zuse-Zentrum für Informationstechnik Berlin
Takustraße 7, D-14195 Berlin-Dahlem
Main lecture room

Abstracts

Małgosia Bednarska-Bzdęga

Degree and Small-graph Avoider-Enforcer games

I will talk on two versions of biased Avoider and Enforcer games played on a hypergraph $\mathcal{H} = (V, E)$. In *the strict Avoider-Enforcer* $(a : f)$ game two players, Avoider and Enforcer, claim in turns previously unselected elements of V, until all vertices are occupied. In each turn Avoider selects exactly a elements, and Enforcer answers by claiming exactly f elements. Avoider loses the game when he claims all elements in at least one edge of the hypergraph; otherwise he wins. The rules of *the monotone Avoider-Enforcer* $(a : f)$ game are almost the same, with the only difference that at every turn Avoider and Enforcer select *at least* a and f elements, respectively.

I will present a winning criterion for Avoider in biased $(a : f)$ Avoider and Enforcer games played on hypergraphs which rank is small if compared with f. Moreover I will discuss *d-degree games* $(1 : f)$, in which Avoider avoids a spanning subgraph of minimal degree at least d in K_n, and *small-graph games* $(1 : f)$, in which Avoider avoids building a copy of a fixed graph G in K_n.

The presented problems continue the research started by D. Hefetz, M. Krivelevich, M. Stojaković, and T. Szabó.

Raman Sanyal

Order preserving maps, polytopes, and applications

Stanley considered the problem of counting order preserving maps from a poset into chains. He showed that many problems in combinatorics relate to this setting and he showed that the counting function – the order polynomial – has many interesting properties. Perhaps the most amazing property is that order polynomials adhere to a combinatorial reciprocity law: Evaluations at negative integers can again be interpreteted in terms of certain order preserving maps. In this talk, I will revisit Stanley's setup and consider the more general problem of counting extensions of partially given order preserving maps. The corresponding counting function is a piecewise polynomial, that is, a collection of polynomials together with a rule which one to evaluate for a given partial map. Like the order polynomial this piecewise polynomial has many interesting properties, among them a generalization of the mentioned combinatorial reciprocity. I will sketch the (royal) route to these results which goes via geometry: Order preserving maps are identified with integer points in polytopes associated to posets. Finally, I will illustrate how counting extensions of order preserving maps can be used to study completions of partial graph colorings and monotone triangles. This is joint work with Katharina Jochemko.

Krzysztof Krzywdziński

A new approach to generalise the Ramsey numbers

The problem is to estimate the number $g(k,l)$. Given positive integers k and l, $g(k,l)$ is the minimum number n such that for any family of graphs $G_1, G_2, \dots G_l$ on n vertices, there are two graphs G_i, G_j, $i \neq j$ such that G_i contains induced subgraph of size k isomorphic to some induced subgraph of G_j. We prove that $g(k,3) = R(k,k)$. We also show some upper and lower bounds for $l > 3$.

Sylwia Antoniuk

Collapse of random group

Let $Z(n,p)$ denote a random group presentation with n generators such that each possible relator of length three is chosen independently with probability p. In the talk we show that the threshold for collapsibility in Żuk's random group model $Z(n,p)$ is smaller than $1.25n^{-3/2}$.

Kolja Knauer

Topological representation of planar partial cubes

Partial cubes are isometric subgraphs of hypercubes. Many graphs are partial cubes. One example are tope graphs of oriented matroids. By the Topological Representation Theorem for Oriented Matroids [Folkman, Lawrence '78] tope graphs may be represented as region graphs of pseudo-sphere arrangements in S^d, but no graph theoretical characterization is known. In the case of rank at most 3 oriented matroids, tope graphs were characterized as planar partial cubes such that for each vertex v there is a unique vertex w such that their distance is the diameter of the graph [Fukuda, Handa '93]. In other words these so-called *antipodal* planar partial cubes correspond to region graphs of pseudo-sphere arrangements in S^2.

Generalizing this result, we present a topological representation theorem for general planar partial cubes in terms of region graphs of arrangements of simple closed curves in S^2 such that every pair of curves intersects in at most 2 points.

Anita Liebenau

On the largest tournament Maker can build

In the k-tournament game, the two players, called Maker and Breaker, alternately claim and direct edges from the complete graph K_n on n vertices. At the beginning of the game, Breaker chooses a goal tournament T_k on k vertices. Maker wins the game if her digraph contains a copy of T_k.

Beck introduced and studied that game. Based on a "random graph intuition" he conjectured that Maker can win the k-tournament game whenever $k \leq (1 - o(1)) \log_2 n$. In fact, we show that Maker has a winning strategy whenever $k \leq (2 - o(1)) \log_2 n$. This is asymptotically tight since for $k \geq (2 - o(1)) \log_2 n$ Breaker can prevent Maker from occupying even just a clique of size k.

Joint work with Dennis Clemens and Heidi Gebauer.

Kasia Rybarczyk-Krzywdzińska

The coupling method for inhomogenious random intersection graphs

Let V be a set of n vertices, $W = \{w_1, \ldots, w_m\}$ be an auxiliary set of m features and $\overline{p} = (p_1, \ldots, p_m)$, $p_i \in (0; 1)$ be a vector. In a general random intersection graph $G(n, m, \overline{p})$ to each vertex v from the vertex set V ($|V| = n$) we assign a set of its features D_v by choosing independently each feature w_i with probability p_i. Then we connect vertices $v, v' \in V$ by an edge if and only if sets D_v and $D_{v'}$ intersect.

We will extend to the general case the coupling method used to obtain sharp threshold functions in $G(n, m, \overline{p})$ with $p_i = p$ for all i. Moreover we will be able to enhance the results obtained by the former approach. In particular we will show how this method applies to get sharp threshold for Hamilton cycle, perfect matching and k-connectivity.

Jan-Philipp Kappmeier

Abstract flows over time

Flows over time generalize classical network flows by introducing a notion of time. Each arc is equipped with a transit time that specifies how long flow takes to traverse it, while flow rates may vary over time within the given edge capacities. We apply this concept of a dynamic optimization problem to the more general setting of abstract flows. In this model, the underlying network is replaced by an abstract system of linearly ordered sets, called "paths"' satisfying a simple switching property: Whenever two paths P and Q intersect, there must be another path that is contained in the beginning of P and the end of Q.

We show that a maximum abstract flow over time can be obtained by solving a weighted abstract flow problem and constructing a temporally repeated flow from its solution. We will see, that the relatively modest switching property of abstract networks already captures many essential properties of classical networks.

This is joint work with Jannik Matuschke and Britta Peis.