EINSTEIN WORKSHOP ON LATTICE POLYTOPES

HARNACK HAUS, BERLIN, 12–15 DECEMBER 2016

Contents

Welcome to the *Einstein Workshop on Lattice Polytopes* funded by the Einstein Foundation Berlin and the project Discretization in Geometry and Dynamics (SFB Transregio 109)

Basic info

Harnack Haus. The workshop takes place in the Goethe-hall of the Harnack Haus, where participants from outside Berlin are also staying. (Check your reservation details, as posted on the workshop web-page).

Address: Ihnestrasse 16–20, 14195 Berlin, Germany. `https://goo.gl/maps/nVd1bmsk34n`. Walk 150 meters southwest from U3 station Thielplatz (next stop after the main FU stop Dahlem Dorf).

Check in time is 3 p.m. Checkout time is 11 a.m. Reception is open 6 am to 8 pm. If you arrive after hours, ring the bell to the left of the main door and the security guard will give you the keys to your room.

Lunch/dinner. People in the participants list have lunch covered by the workshop, Monday to Thursday, at the Harnack Haus restaurant. Other attendants can have lunch at the "**Einstein Lounge**" of the Harnack Haus (same building, different place; 6.30 EUR for a single dish, 9.50 for starter+entrée+dessert, closed on Wednesday). Alternatively, they can go to the FU mensa (about 8 min walk) or elsewhere.

On Tuesday, participants have a joint dinner at the Harnack Haus before the problem session. The rest of the evenings if you don't want to move much there are several places for dinner in the Dahlem Dorf area (20 min walking or one U-bahn stop north-east from Harnack Haus).

Special sessions. Apart of the 17 lectures, there is the following sessions where you can contribute. Contact one of the organizers if you are interested:

- On Monday afternoon there is a "short presentations" session where people can bring up problems they would like to discuss during the week, make research announcements, etc. Poster presenters will be given the opportunity to briefly describe their posters during this session. Presenters in this session can use the blackboards or bring a file to use the projector.
- On Tuesday after dinner there will be a problem session.

Social activities. The following additional activities are (half)-planned. You do not need to register, just show up (maybe tell the contact person):

- On Sunday evening many of us will get together at Luise (Königin-Luise-Strasse 42, 200m East of the Dahlem Dorf station on the U3 line. `https://goo.gl/maps/ch2cmiVX3kw`). **Time: 7 pm**.
- Wednesday afternoon is free, and some people will go ice-skating after lunch. Details will be given during the workshop. Contact: Christian Haase. **Estimated departure time from HH: 2 pm**.
- On Thursday evening something will be organized, which will probably include salsa dancing. More details will be given during the workshop. Contact: Mónica Blanco.

List of participants

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Program

	Monday 12th	Tuesday 13th	Wednesday 14th	Thursday 15th
08:30–08:50	Pick up your tag			
08:50–09:00	Welcome words			
09:00–09:45	**Winfried Bruns**	**Achill Schürmann**	**Jesús De Loera**	**Jeff Lagarias**
09:45–10:15	Coffee	Coffee	Coffee	Coffee
10:15–11:00	**Matthias Henze**	**Johannes Hofscheier**	**Stefan Weltge**	**Federico Castillo**
11:15–12:00	**Ivan Soprunov**	**Akihiro Higashitani**	**Gennadiy Averkov**	**Katharina Jochemko**
13:00–14:00	Lunch	Lunch	Lunch	Lunch
14:45–15:30		**Poster session**		
15:30–16:00	Coffee	Coffee		Coffee
16:00–16:45	**Alexander Kasprzyk**	**Thomas Kahle**		**Jan Hofmann**
17:00–17:45	**Short presentations**	**Raman Sanyal**		**Michael Joswig**
17:45–18:30	**Short presentations**			
18:30–20:00		Conference Dinner		
20:00–21:30		Problem Session		

Abstracts of talks

Gennadiy Averkov, Minkowski-type theorems for lattice polytopes

I will report on the recent progress in the study of lattice polytopes with exactly one interior lattice points.

Winfried Bruns, Convex hull computation in Normaliz

Normaliz is a versatile package for computations in discrete convex geometry: it computes lattice points in polyhedra, or, from a different perspective, solves linear diophantine systems of inequalities, equations and congruences.

Recently it has been included in benchmarks of convex hull computation (or vertex enumeration), and has often outperformed dedicated systems, especially on large problems. We will present Normaliz' approach to convex hulls. The main tool is pyramid decomposition whose development was sparked off by the more difficult task of computing triangulations. Pyramid decomposition allows "localization" and helps to break the often forbidding complexity of pure Fourier-Motzkin elimination. Furthermore it is parallelization friendly.

Based on: W. Bruns, B. Ichim and C. Söger, The power of pyramid decomposition in Normaliz. J. Symb. Comp. 74 (2016), 513–536.

Federico Castillo, Ehrhart positivity

We call a polytope P positive, if its Ehrhart polynomial has all positive coefficients. In the literature there are different examples using different techniques to prove positivity. We will survey some of these results. It turns out, through work of Danilov, McMullen, and others, that there is an interpretation of the coefficients relating to the normalized volume of faces. We try to make this relation more explicit in the particular case of the regular permutohedron. The goal, long term, is to prove positivity for all generalized permutohedra. This is joint work with Fu Liu.

Jesús De Loera, Tverberg-style theorems over lattices and other discrete sets

This year we celebrate 50 year of the lovely theorem of Helge Tverberg!

Let $a_1, \ldots, a_n$ be points in $\mathbb{R}^d$. If the number of points n satisfies $n > (d+1)(m-1)$, then they can be partitioned into m disjoint parts $A_1, \ldots, A_m$ in such a way that the m convex hulls $\operatorname{conv} A_1, \ldots, \operatorname{conv} A_m$ have a point in common.

Over the years many generalizations and extensions, including colorful, fractional, and topological versions, have been developed and are a bounty for discrete geometers. My talk will discusses yet another fascinating way to interpret Tverberg's theorem, now with a view toward number theory, lattices, integer programming, all things discrete not continuous nor topological.

Given a discrete set S of $\mathbb{R}^d$ (e.g., a lattice, or the Cartesian product of the prime numbers), we study the number of points of S needed to guarantee the existence of an m-partition of the points $A_1, \ldots, A_m$ such that the intersection of the m convex hulls of the parts contains at least k points of S. The proofs of the main results require new quantitative integer versions of Helly's and Carathéodory's theorems.

This is joint work with Reuben La Haye, David Rolnick, and Pablo Soberón.

Matthias Henze, On the covering radius of lattice polytopes and its relation to view-obstructions and densities of lattice arrangements

The covering radius of a convex body K in $\mathbb{R}^n$ is the minimal dilation factor $r > 0$ such that the lattice arrangement $rK + \mathbb{Z}^n$ covers the whole space $\mathbb{R}^n$. This is a classic concept in the geometry of numbers and has been of interest ever since its introduction. We study the covering radius on the class of lattice zonotopes and in particular provide upper bounds depending on the number of line segments that generate the zonotope. Moreover, we exemplify that the exact determination of this parameter, even for very structured lattice simplices, is a highly non-trivial task. In the talk, we also explain how these findings relate to view-obstruction problems (in the case of lattice zonotopes), and to questions on minimal densities of lattice arrangements (in the case of lattice simplices).

This is based on joint work with Romanos-Diogenes Malikiosis (arxiv.org/abs/1609.01939) and Bernardo González Merino (arxiv.org/abs/1605.00443).

Akihiro Higashitani, Lattice simplices of maximal dimension with a given degree

It was proved by Nill that for any lattice simplex of dimension d with degree s which is not a lattice pyramid, the inequality $d + 1 \leq 4s - 1$ holds, while this bound is not sharp in general. In this talk, after providing a sharp bound for this inequality, we will give a classification of lattice simplices attaining the equality.

Jan Hofmann, The finiteness threshold width of lattice polytopes

Trying to classify lattice polytopes, it seems natural to try to enumerate all of them by fixing the dimension and the number of lattice points. In this talk we will see this works marvelously in dimensions 1 and 2, but fail miserably from dimension 3 on. One possible remedy for this is the finiteness threshold width of lattice polytopes, which is defined as the width $w^\infty(d)$ beyond which there are only finitely many d-dimensional lattice polytopes with a given number of lattice points and width $> w^\infty(d)$. In this talk we will see that $w^\infty(d)$ is finite and $w^\infty(4) = 2$.

This is joint work with Mónica Blanco, Christian Haase and Francisco Santos.

Johannes Hofscheier, Lattice simplices of bounded degree.

In this talk we present a"moduli"-approach for the study of lattice simplices of bounded degree s. More precisely we introduce a topology on the space of lattice simplices of bounded degree and suggest a compactification of it by relating simplices to certain subgroups of the real euclidean space. Using Chabauty-Pontryagin duality we can show that these subgroups form only finitely many maximal families. We present a classification of these maximal families for the bound $s = 2$ and conclude the talk by applying our results to the description of the h^*-vectors of degree 2 lattice simplices.

This is joint work with Akihiro Higashitani.

Katharina Jochemko, Real-rooted h^*-polynomials.

Interlacing polynomials are a powerful tool to attack real-rootedness questions. In this talk I will explain the method and present applications to h^*-polynomials of certain families of lattice polytopes.

Michael Joswig, Matroids From Hypersimplex Splits

Studying regular subdivisions of hypersimplices naturally leads to a new class of matroids, which we call split matroids. Very many matroids are of this type; e.g., the paving matroids arise as special cases. It turns out that the structural properties of the split matroids can be exploited to obtain new results in tropical geometry, especially on the rays of the tropical Grassmannians.

Joint work with Benjamin Schröter.

Thomas Kahle, The geometry of rank one tensor completion.

Rank-one tensors form a well understood toric variety. We study the geometry of restrictions of this variety to subsets of the coordinates. This gives insight into the problem of rank-one completion of partial tensors. Over an algebraically closed field, this tensor completion leads to questions in lattice point combinatorics. Particular emphasis is also put on the semialgebraic nature of the problem, which arises for real tensors with constraints on the parameters. The algebraic boundary of the completable region is described for tensors parametrized by probability distributions and where the number of observed entries equals the number of parameters. If the observations are on the diagonal of a tensor of format $d \times \cdots \times d$, the complete semialgebraic description of the completable region is found.

Alexander Kasprzyk, The combinatorics of Mirror Symmetry

Recent progress in understanding Mirror Symmetry for Fano manifolds by Coates, Corti, and myself is closely connected with the combinatorics of lattice polytopes. Although many of the motivating ideas remain conjectures, this new view-point is making significant and unexpected contributions to both the theory of lattice polytopes, and to how we may approach the classification of Fano varieties. I hope to sketch these connections, justify what we expect in higher dimensions, and explain some of the main challenges.

Jeffrey Lagarias, Dilated floor functions and their commutators

This talk concerns interactions of the discrete and continuous. A dilated floor function is $f_a(x) = \lfloor ax \rfloor$, where $a \in \mathbb{R}$ is a real dilation factor. Multivariate analogues $\lfloor a_0 + a_1 x_1 + ... + a_n x_n \rfloor$ appear in sieve methods, in counting lattice points in various regions, and in computational geometry in discretizing hyperplanes. This talk considers interactions between dilated floor functions having two different dilation scales, call them $f_a(x)$ and $f_b(x)$. In general such functions do not commute under composition, and we study behavior of their commutators. In joint work with Takumi Murayama and D. Harry Richman, we determine all parameter pairs (a, b) where the functions commute. Further work with D. Richman studies the more general problem of determining all pairs (a, b) where a one-sided commutator inequality holds for all x. The latter problem has unexpected connections to Beatty sequences and to the two-dimensional Diophantine Frobenius problem.

Raman Sanyal, Combinatorially mixed valuations on polytopes

The confluence of Minkowski addition and volume gives rise to the vast theory of mixed volumes. A fundamental result is that mixed volumes are linear, nonnegative, and monotone. In the discrete setting (i.e. lattice polytopes), discrete volume (i.e. counting lattice points) takes the place of the volume. The resulting discrete mixed volumes are linear by construction but nonnegativity and monotonicity is genuinely lost. In this talk I will advertise the notion of combinatorially mixed valuations associated to valuations on (lattice) polytopes. For general polytopes, the theory of combinatorially mixed valuations parallels that of mixed valuations. For lattice polytopes, many combinatorially mixed valuations (including the combinatorial mixed volume) are nonnegative and monotone and the study of general combinatorially mixed valuations is strongly tied to the combinatorics of subdivisions of lattice polytopes.

This is joint work with Katharina Jochemko.

Achill Schürmann, Exploiting Symmetries of Lattice Polytopes

Exploiting symmetry in integer linear programming and lattice point counting are two difficult problems for which no good general approach exists. In fact, standard techniques work particularly poor on symmetric problems. In this talk we give an overview about ongoing work on new symmetry exploiting techniques for these two fundamental problem classes involving lattice polytopes. We in particular present some new ideas of decomposing lattice polytopes and give some initial proof-of-concept results applying these new techniques.

Ivan Soprunov, Minkowski length of lattice polytopes.

Let P be a lattice polytope. Its Minkowski length $L(P)$ is defined as the largest number of lattice segments whose Minkowski sum is contained in P. If we require the segments to be collinear we get the definition of the lattice diameter of P, so the Minkowski length is a natural extension of this notion. The definition comes from studying the number and the structure of irreducible factors of polynomials in the space of Laurent polynomials with a given Newton polytope. I will talk about the behavior of $L(tP)$ as a function of the scaling factor $t \in \mathbb{N}$. In a recent joint work with Jenya Soprunova we showed that $L(tP)$ is eventually a quasi-polynomial with linear constituents.

Stefan Weltge, Polytopes in the 0/1-Cube with Bounded Chvátal-Gomory Rank

Let $S \subseteq \{0,1\}^n$ and R be any polytope contained in $[0,1]^n$ with $R \cap \{0,1\}^n = S$. We prove that R has bounded Chvátal-Gomory rank (CG-rank) provided that S has bounded pitch and bounded gap, where the pitch is the minimum integer p such that all p-dimensional faces of the 0/1-cube have a nonempty intersection with S, and the gap is a measure of the size of the facet coefficients of $\operatorname{conv}(S)$.

Let $H[\bar{S}]$ denote the subgraph of the n-cube induced by the vertices not in S. We prove that if $H[\bar{S}]$ does not contain a subdivision of a large complete graph, then both the pitch and the gap are bounded. By our main result, this implies that the CG-rank of R is bounded as a function of the treewidth of $H[\bar{S}]$. We also prove that if S has pitch 3, then the CG-rank of R is always bounded. Both results generalize a recent theorem of Cornuéjols and Lee (IPCO 2016), who proved that the CG-rank is always bounded if the treewidth of $H[\bar{S}]$ is at most 2.

This is joint work with Yohann Benchetrit, Samuel Fiorini, and Tony Huynh.

List of posters

Óscar Iglesias, Classification of lattice empty 4-simplices

Gabriele Balletti, On the maximum dual volume of a canonical Fano polytope

Jorge Alberto Olarte, Rational Harnack Curves on Toric Surfaces

Mónica Blanco, Enumerating lattice 3-polytopes

Camilo Sarmiento, Geometry of ν-Tamari lattices in types A and B

Christoph Pegel, Generic Marked Poset Polytopes