Einstein Workshop on Geometrical and Topological Combinatorics

Titles, abstracts, and posters

October 29 to Octover 31 2018, Freie Universiät Berlin

1 Talks

1.1 Cesar Ceballos

Title: *Hopf Dreams*

Pipe dreams are certain combinatorial objects that play a fundamental role in the combinatorial understanding of Schubert polynomials. They encode many remarkable geometric structures such as associahedra, multiassociahedra, and certain polytopal subdivisions realizing the v-Tamari lattices of Préville-Ratelle and Viennot. In this talk, I will describe a Hopf algebra structure on a family of pipe dreams. This Hopf algebra gives rise to intriguing connections to the enumeration of certain lattice walks on the quarter plane and applications to the theory of multivariate diagonal harmonics. This is joint work with Nantel Bergeron and Vincent Pilaud.

1.2 Clément Maria

Title: *Treewidth, crushing, and hyperbolic volume*

The treewidth of a 3-manifold triangulation plays an important role in algorithmic 3-manifold theory, and so it is useful to find bounds on the treewidth in terms of other properties of the manifold. In this talk, we prove that there exists a universal constant c such that any closed hyperbolic 3-manifold admits a triangulation of treewidth at most the product of c and the volume. The converse is not true: we show there exists a sequence of hyperbolic 3-manifolds of bounded treewidth but volume approaching infinity. Along the way, we prove that crushing a normal surface in a triangulation does not increase the carving-width, and hence crushing any number of normal surfaces in a triangulation affects treewidth by at most a constant multiple. This is joint work with Jessica Purcell.

1.3 Arnau Padrol

Title: *On Moser's shadow problem*

In a famous list of problems in combinatorial geometry from 1966, Leo Moser asked for the largest $s(n)$ such that every 3-dimensional convex polyhedron with n vertices has a 2-dimensional shadow with at least s(n) vertices. I will describe the main steps towards the answer, which is that $s(n)$ is of order $\log(n)/\log(\log(n))$, found recently in collaboration with Jeffrey Lagarias and Yusheng Luo, and which follows from 1989 work of Chazelle, Edelsbrunner and Guibas. I will also report on current work with Alfredo Hubard concerning higher-dimensional generalizations of this problem.

1.4 Joao Paixao

Title: *Discrete Line Fields on Surfaces*

A line field on a surface is a smooth map that assigns a tangent line to all but a finite number of points. Such fields model a number of geometric and physical properties, for example the principal curvature directions on surfaces or the stress flux in elasticity. They can be seen as a generalization of vector fields. In this talk, we provide a purely combinatorial definition of line fields, the discrete line fields, which generalizes Formans discrete vector fields and all the analogous results from discrete Morse theory.

1.5 José Samper

Title: *Finiteness theorems for matroid complexes with prescribed homotopy type*

It is well known that the independence complex of any matroid without coloops is homotopy equivalent to a wedge of $k > 0$ equidimensional spheres. We prove that if the dimension and the number of spheres is fixed, then only finitely many such independence complexes exist. This counterintuitive property leads to new structural questions such as upper and lower bound theorems/conjectures for matroids based on the two parameters mentioned. It also leads natural to new theorems and questions about face numbers of such complexes. If time permits we will discuss similar results for geometric lattices and conjectures for broken circuit complexes. This is joint work with F. Castillo.

1.6 Matthias Schymura

Title: *On compact representations of Voronoi cells of lattices*

In a breakthrough work for computational geometry of numbers, Micciancio and Voulgaris (2010) described a single exponential time deterministic algorithm for the Closest Vector Problem on lattices. Their algorithm is based on the computation of the Voronoi cell of the input lattice and thus may need exponential space as well. Motivated by the question whether there exists such an algorithm that needs only polynomial space, we introduce and study the concept of compact lattice bases: A lattice basis is c-compact if every facet normal of the Voronoi cell is an integral linear combination of the basis vectors using coefficients that are bounded by c in absolute value. In the talk, we will see that (1) there always exist c-compact bases, where c is a polynomial in the rank of the lattice; (2) there are lattices without a c-compact basis with c growing sublinearly with the rank; and (3) that every lattice with a zonotopal Voronoi cell has a 1-compact basis. These results are drawn from transference results in the geometry of lattices, and from the rich combinatorial structure of zonotopes. This is joint work with Christoph Hunkenschröder and Gina Reuland.

1.7 Kristin Shaw

Title: *Chern-Schwartz-MacPherson classes of matroids*

Chern-Schwarz-Macpherson (CSM) classes are one way to extend the notion of Chern classes of the tangent bundle to singular and non-complete algebraic varieties. In this talk, I will provide a combinatorial analogue of CSM classes for matroids, motivated by the geometric case of hyperplane arrangements. The CSM classes of matroids are polyhedral fans which are Minkowski weights. One goal for defining these classes is to express matroid invariants using the language of algebraic geometry and in turn use geometric intuition to study the properties of these invariants. Moreover, CSM classes can be used to study the complexity of more general objects such as subdivisions of matroid polytopes and tropical manifolds. This is based on joint work with Lucia López de Medrano and Felipe Rincón.

1.8 Jacinta Torres

Title: *Generalizing LS galleries in affine buildings*

In 2005, Gaussent-Littelmann have interpreted LS galleries in terms of the geometry of affine grassmannians. I will present a generalization of this notion for any Littelmann model of galleries by keeping careful track of the load bearing walls of these galleries. Finally I will present an interpretation of these load-bearing walls in terms of retractions. This is joint work in progress with Petra Schwer.

1.9 Hailun Zheng

Title: *How to triangulate the product of spheres?*

Finding the vertex-minimal triangulation of a manifold is an important yet hard problem in computational topology. The minimal triangulation of $\mathbb{S}^i \times \mathbb{S}^{d-i-1}$ can be found in lower dimensional cases by computer search. Besides that very little is known: Kühnel constructed the minimal triangulation of $\mathbb{S}^1 \times \mathbb{S}^{d-2}$. More recently Klee and Novik found a small centrally symmetric triangulation of $\mathbb{S}^i \times \mathbb{S}^{d-i-1}$ as a subcomplex of the octahedral d-sphere for all i.

In this talk we will survey the results on the triangulations of sphere products and discuss the main ideas behind several constructions. Then we will propose new balanced or centrally symmetric triangulations of $\mathbb{S}^i \times \mathbb{S}^{d-i-1}$ with few vertices. The construction is inspired by the handle theory.

Joint work with Alexander Wang.