Babyseminar, Winter Semester 2013
Class Field Theory

1, 2-Local CFT: Basic materials - 15/10/2013 , 22/10/2013 : The basic notions for this section can be founded in most of the classical literature. You may use for example Prof. Esnault's notes on Algebraic Number Theory II. The essential topics to be covered are: definition of absolute value, nonarchimedean absloue values, discrete vaulation rings, completion, Newton's lemma and some examples of its application, definition of local fields, the notion of ramification index and residue fields, the "efg=n" theorem, ramified and unramified extensions, the classification of ramified, totally ramified and unramified extensions over local fields, e.g. the Eisenstein polynomials and Kummer extensions. We also need some material about global fields like decomposition groups and discriminant of an extension. For this part a good and brief source could be [p. 134-143 ANT]. Since the time is limited, we suggest to the speaker to give a good idea of the theory via examples, skipping the proofs. Good examples may be: example of extensions of p-adic fields, computing the ramification index of a simple extension like $\mathbb{Z} \to \mathbb{Z}[i]$ and so on.

3,4-Local CFT: The main theorems of the local CFT - 29/10/2013,5/11/2013: State the existence and the reciprocity theorem. Specific notes will be provided for these two lectures.

5-Local CFT: The cohomology of groups - 12/11/2013: Go through pages 55-66 [CFT] until the paragraph "Functorial properties of the cohomology groups".

6-Local CFT: Homology and Tate's groups - 17/11/2013: Cover pages 66-86 [CFT] until the Appendix. Tate's theorem is not needed.

7-Local CFT - Cohomological approach to local CFT-19/11/2013: Go through the pages 95-105 [CFT] until the Local Artin map.

8-Local CFT Local Artin map and Existence theorem-26/11/2013: State the Tate's theorem without proving it and cover pages 105-115 [CFT] as deep as time permits.

9-Local CFT: Brauer group - 7/1/2014: Define the Brauer group (additional notes for this part will be provided), try to cover the pages 132-136 [CFT], from "Central Simple Algebras and 2-cocyles" until "A nonarchimedean local field".

10-Global CFT: introduction- 14/1/2014: Define the Frobenius element (p.148 [CFT]), the global Artin map (p.152 [CFT]), introduce and define the notion of topological groups, the idèles and the idèle class group (p.165 and ff [CFT]). Define the norms on idèles (p.172 [CFT])

11-Global CFT: The main theorems - 21/1/2014: Go through the statement of the two main theorems (V.5 [CFT]) and give an outline of the cohomological theory of the idèles (VII.2 [CFT]). If time permits give the outline of the proof of the theorems (VII.1 [CFT]).

12-Global CFT: Proofs (1) - 28/1/2014: State and prove the first inequality (VII.4 [CFT]). State the second inequality. Recall the reciprocity theorem and explain why it is implied by Thm VII 8.1 [CFT]

13-Global CFT: Proofs (2) - 4/2/2014: Prove Thm VII 8.1 [CFT]. Recall the existence theorem and prove it (VII.9).

14-Global CFT: Applications - 11/2/2014: There are some interesting applications, you may pick some between: Brauer groups (Thm VII 7.1 [CFT]), Local-global principle for norms and quadratic forms (VIII.3 [CFT]), Classification of quadratic forms over number fields (VIII.6 [CFT]), or choose others.

[CFT] J.S. Milne. *Class field theory (v4.02)* `http://www.jmilne.org/math/CourseNotes/CFT.pdf`

[ANT] J.S. Milne. *Algebraic Number Theory (v3.05)* `http://www.jmilne.org/math/CourseNotes/ANT.pdf`