December 17, 2013

Algebra I

Prof. H. Esnault

Exercise sheet 10[1]

Exercise 1. Which of the following rings are noetherian? Prove your claims.

(a) The ring of continuous functions $[0, 1] \to \mathbb{R}$
(b) The ring of functions $\mathbb{N} \to \mathbb{Z}/2\mathbb{Z}$.
(c) A subring R of $k[x]$ containing k, where k is a field.

Exercise 2. Let A be a ring and M and A-module. Show that the following statements are equivalent:

(a) M is noetherian.
(b) Every non-empty set of submodules of M has a maximal element.
(c) Every non-empty set of *finitely generated* submodules of M has a maximal element.

Exercise 3. Is a subring of a noetherian ring noetherian? If yes, prove it. If not, find a counterexample.

Exercise 4. Let A be a ring and M a noetherian A-module. Let $\phi : M \to M$ be an endomorphism. Prove that $\ker(\phi^n) \cap \operatorname{im}(\phi^n) = 0$ for large n. From this conclude that ϕ is an isomorphism if and only if ϕ is surjective (which you already proved in a more general context on problem set 3).

[1] If you want your solutions of this exercise to be corrected, please hand them in just before the lecture on January 7. Questions or comments to kindler@math.fu-berlin.de or come to A3, Room 112A.