Coupled Fluid Flow

Darcy‘s Law

$$\vec{q}_M = \frac{k\rho}{\mu}(\nabla p - \vec{g}\rho)$$ [kg m/s]

$$\vec{q}_D = K\nabla(p - g\bar{\rho}z)$$ [m/s]

$$\vec{q}_h = K^*\nabla(h)$$ or $$\vec{q}_h = K^*\frac{\Delta h}{\Delta l}$$ or′ $$\vec{q}_D = K(\nabla p/\gamma + z)$$ [m/s]

$$\frac{\partial \varphi\rho}{\partial t} \pm Q = \nabla(\frac{k\rho}{\mu}(\nabla p - \vec{g}\rho))$$

$$S\frac{\partial h}{\partial t} = \nabla(\frac{k}{g\mu}\nabla(h))$$

Heat conduction

$$\vec{j}_T = -\lambda \nabla T$$

The change of energy within the control volume becomes

$$\frac{\partial c \rho T}{\partial t} = \nabla(\lambda \nabla T) \quad \text{[J/s]}$$

, Or if sources or sinks are vailable (chemical reactions, radioactive heat production)

$$\frac{\partial c \rho T}{\partial t} \pm Q = \nabla(\lambda \nabla T)$$

Observe that $c = c(T)$ and $\rho = \rho(T)$ and that the specific volume will also depend on the temperature. In detail the size of the control volume changes

Dimensions
of
Energy

(amount of heat, work)

$$1J = 1\frac{kg\ m^2}{s^2} = 1Nm$$

$$= 1\ Ws$$

Units

$Heat\ flow\ q$: Wm^{-2}

$conductivity\ \boldsymbol{\lambda}$ (k): W$m^{-1}K^{-1}$

$diffusivity\ \boldsymbol{\kappa}$: m^2s^{-1}

$specific\ heat\ c_p$: $J\ kg^{-1}K^{-1}$

$heat\ production$: W m^{-3}
$\mu W/m^3$

Fick‘s 1st law

$$\vec{j}_c = -D\nabla C$$

D is mostly given as the diffusion coefficent in pure water. Provide an estimate for porous media.

The flux is related to the temporal change by

$$\frac{\partial \phi C}{\partial t} = \nabla(D\nabla C)$$

Or if chemical reactions are involved within the porous medium

$$\frac{\partial \phi C}{\partial t} \pm Q = \nabla(D\nabla C)$$

D: $m^2 s^{-1}$

Coupled equations

$$\frac{\partial \varphi C}{\partial t} - \nabla(D(\nabla C)) + \nabla(C\vec{q}) + Q_C = 0$$

$$\frac{\partial c_b \rho_b T}{\partial t} - \nabla(\lambda(\nabla T)) + \nabla(c_f \rho_f T\vec{q}) + Q_T = 0$$

$$\frac{\partial \varphi \rho_f}{\partial t} + \nabla \vec{q} + Q_f = 0$$

$$\vec{q} = -\frac{k\rho}{\mu}(\nabla p - \vec{g}\rho)$$

$$\varphi = \varphi(p, T, Q_c)$$

D=D(T)

$$c_b = c_b(p, T, Q_c)$$

$$c_f = c_f(p, T, Q_c)$$

$$\rho_f = \rho_f(p, T, Q_f)$$

$$\rho_b = \rho_b(p, T, Q_f, Q_c)$$

$$\lambda(T)$$

$$k(p, T.Q_i)$$

Mathematical formulation of the thermohaline flow problem in FEFLOW

$$\mathrm{S}_0 \frac{\partial \varphi}{\partial \mathrm{t}} + \mathrm{div}(\mathbf{q}) = Q_{\mathrm{Boussinesq}}$$

$$\mathbf{q} = -\mathbf{K}\left(\mathbf{grad}(\varphi) + \frac{\rho_f - \rho_{0f}}{\rho_{0f}} \right)$$

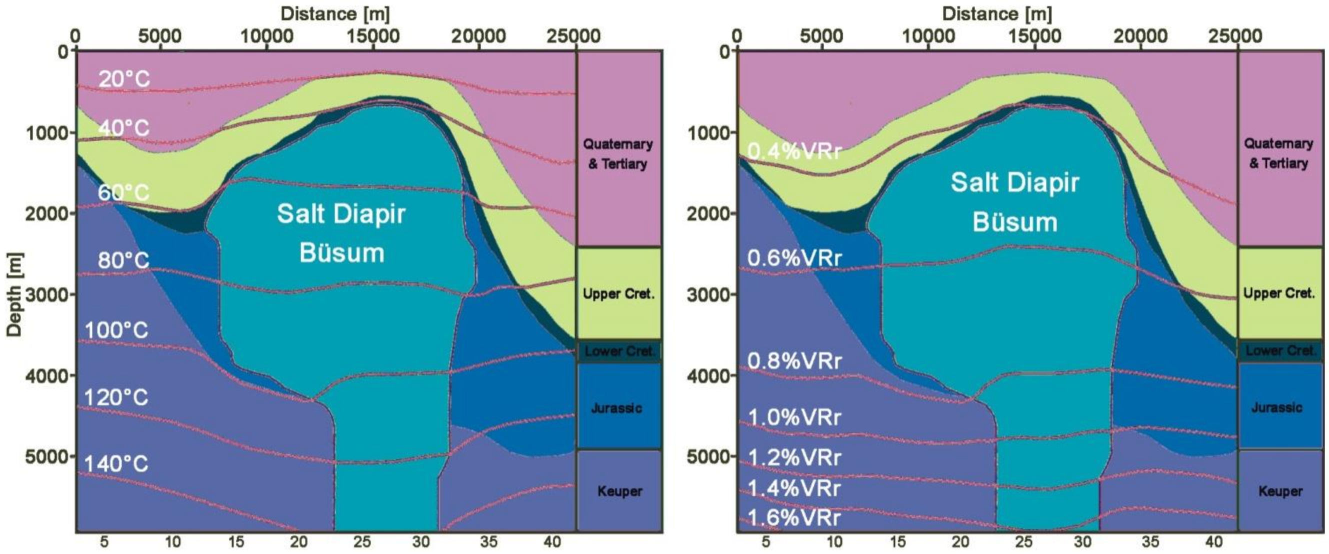
$$\frac{\partial \phi C}{\partial t} + \mathrm{div}\left(\mathbf{q}C\right) - \mathrm{div}\left(\mathbf{Dgrad}(C)\right) = Q_C$$

$$\frac{\partial}{\partial t}\left(\left(\phi\rho_f c_f + (1-\phi)\rho_s c_s\right)T\right) + \mathrm{div}(\rho_f c_f T\mathbf{q}) - \mathrm{div}\left(\boldsymbol{\lambda}\mathbf{grad}(T)\right) = Q^T$$

$$\mathbf{K} = \frac{\mathbf{k}\rho_{0f}\, g}{\mu_f(C,T)}$$

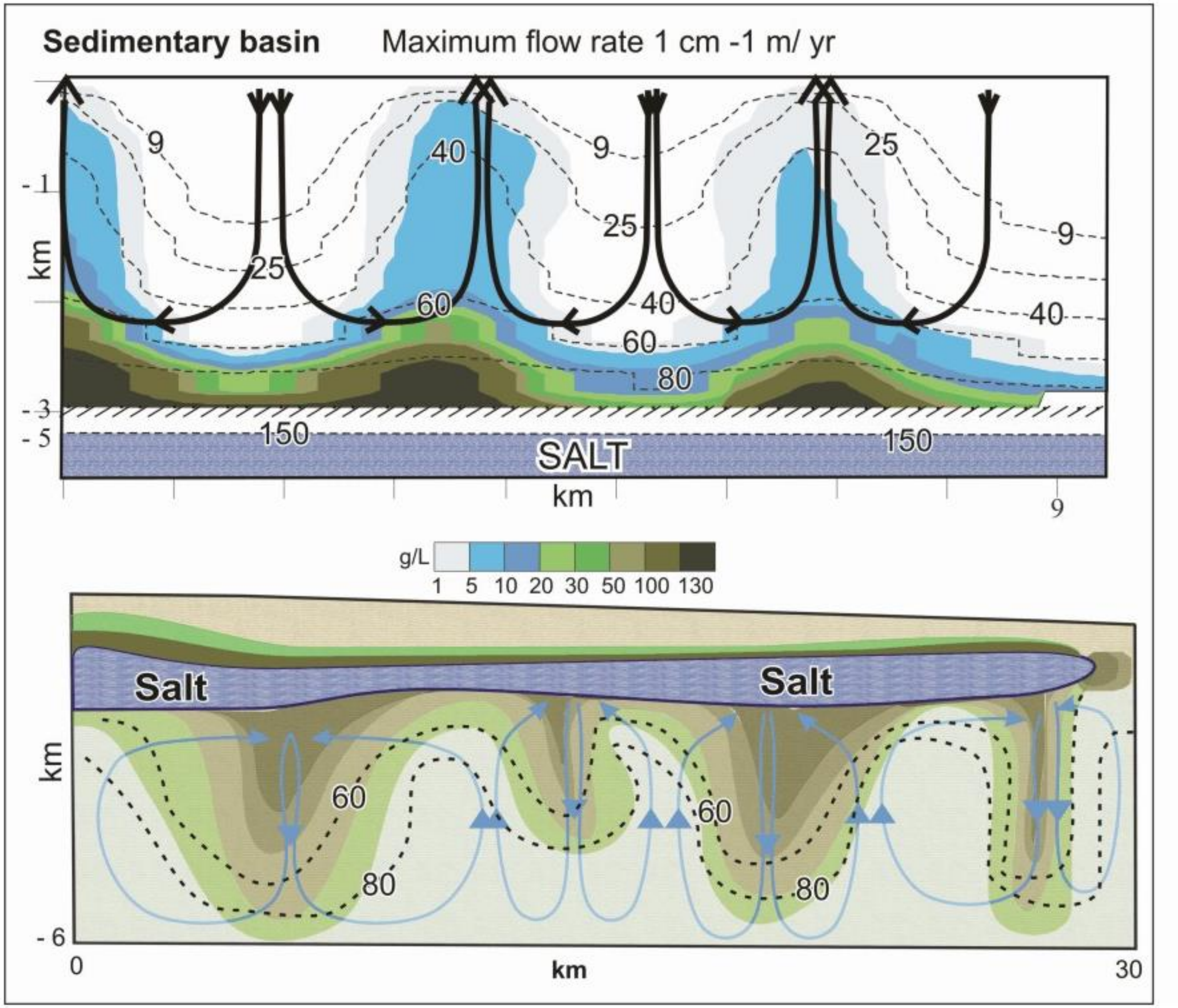
$$\rho^f = \rho_0^f \left(1 - \overline{\beta}(T,p)(T - T_0) + \overline{\gamma}(T,p)(p - p_0) + \frac{\overline{\alpha}}{C_{sat} - C_0}(C - C_0) \right)$$

$$\overline{\alpha} = \frac{\rho_{sat}^f - \rho_0^f}{\rho_0^f}$$



Cross section through the Büsum diapir and adjacent rim synclines with temperature isolines (left) and vitrinite reflectance isolines (right)

Type of thermohaline flow

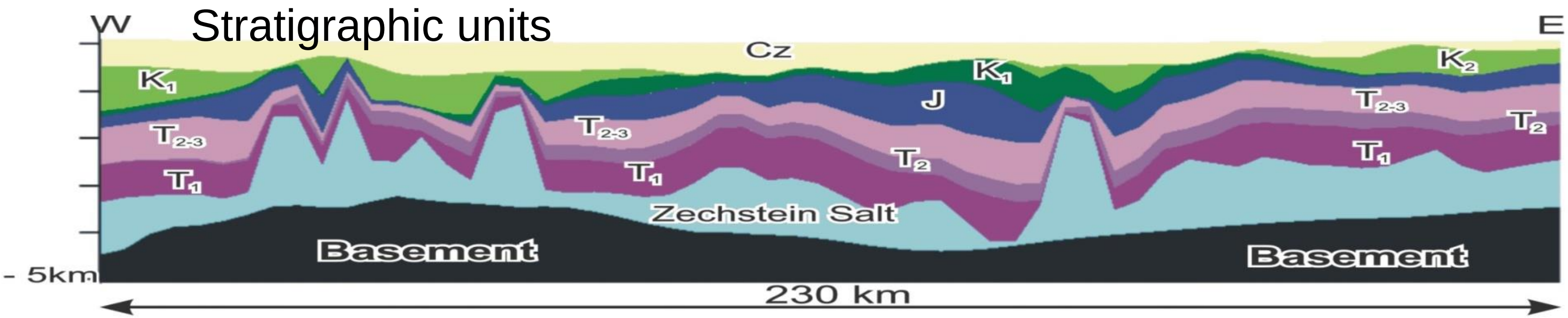


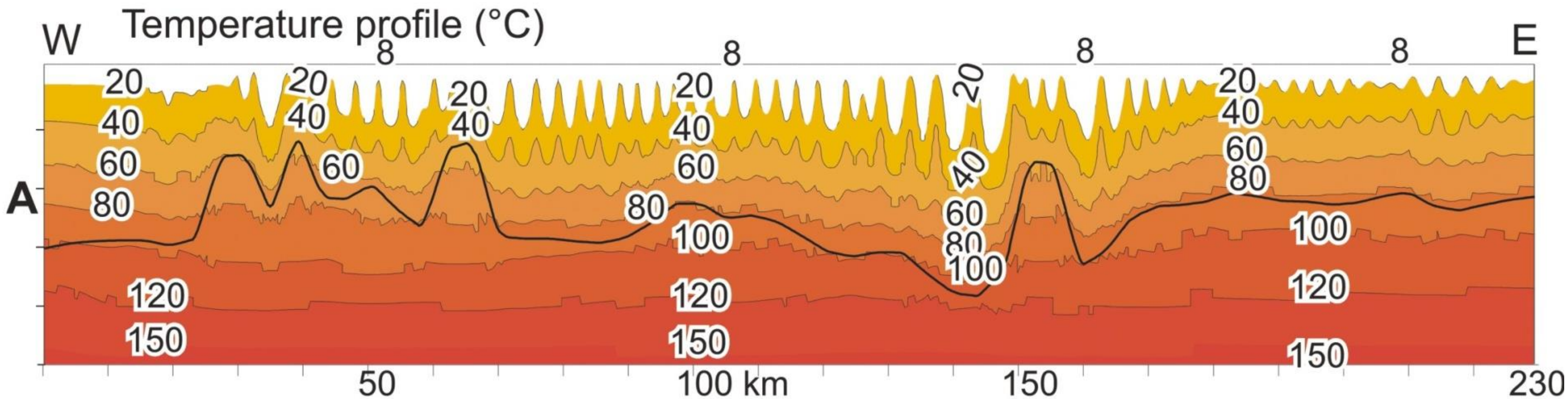
Thermally induced brine plumes developping on a deep salt sheet

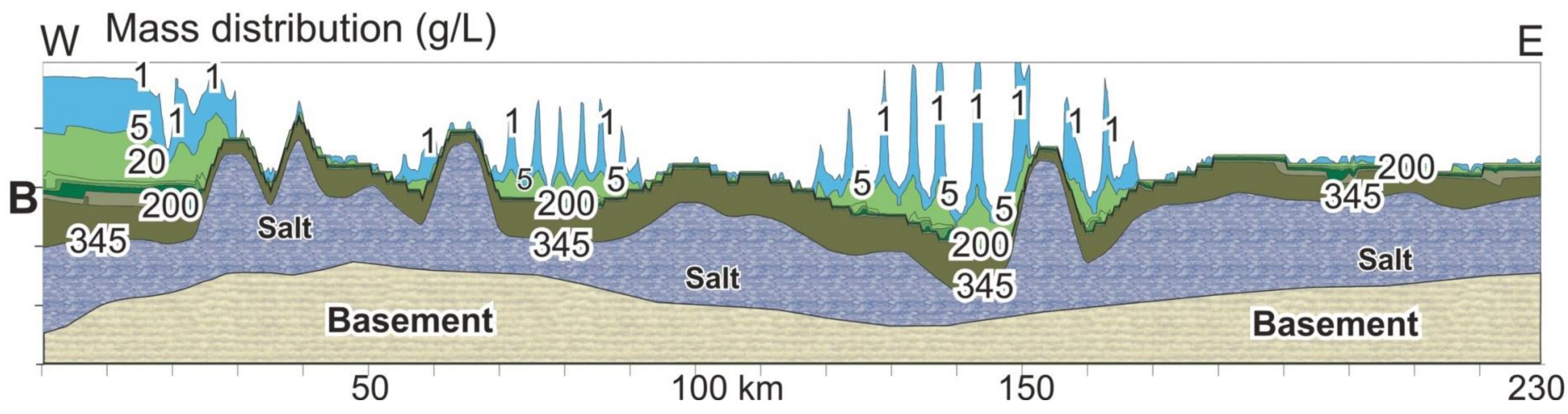
Brine lenses, gravitational convection from a shallow salt sheet

Dasehd lines: isotherms (°C)

Thermohaline flow in the NE German Basin







Stability criteria

Thermal Rayleigh number

$$Ra_T = \frac{K\bar{\beta}\Delta Td}{\Lambda}$$

Solutal Rayleigh number

$$Ra_s = \frac{\frac{\bar{\alpha}}{C_{sat} - C_0} K\Delta Cd}{\varepsilon D_d}$$

The solutal and thermal Rayleigh numbers are related by

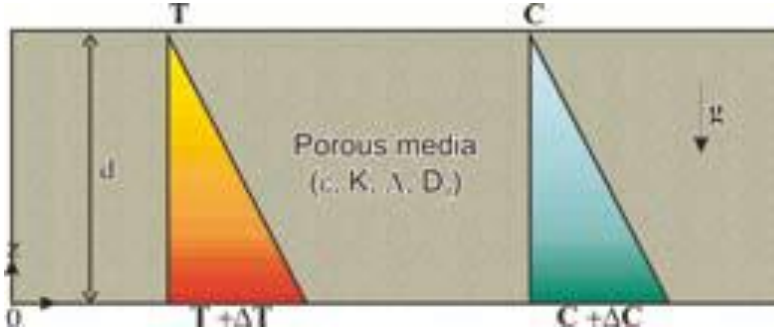
$$Ra_s = N \times Le \times Ra_T$$

Buoyancy ratio (Turner)

$$N = \frac{\frac{\bar{\alpha}}{C_{sat} - C_0}\Delta C}{\bar{\beta}\Delta T}$$

Lewis number

$$Le = \frac{\Lambda}{\varepsilon D_d}$$



•**The monotonic instability (or stationary convection) boundary is a**
•**straight line defined by**

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$$Ra_C = Ra_T + Ra_s = 4\pi^2$$

Ra_C is the critical Rayleigh number.

The region delimited by $Ra_T + Ra_s < 4\pi^2$

is a stable regime characterized by pure conduction and no convection.

In a range between $4\pi^2 < Ra_T + Ra_s < 240 - 300$

steady state convective cells develop

For $Ra_T + Ra_s > Ra_{c2}$ the convection regime is unstable