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AG Mathematical Geometry Processing

Differential Geometry I
Winter Semester 2025/2026
Freie Universität Berlin

Exercise Sheet 7

Version 1, Published: 1.12.2025

Submission: 8.12.2025, 8:30 AM (start of the tutorial) or 10:15 AM (start of the lecture)

Exercise 1. (4 points)

Describe the initial value problem for geodesics on a torus

$$f : (u, v) \mapsto (\cos(u)(R + r\cos(v)), \sin(u)(R + r\cos(v)), r\sin(v)), \quad 0 < r < R. \tag{1}$$

Use the differential equations to verify that the following curves on the torus are geodesics:

i) $t \mapsto f(u_0, t)$ for a constant u_0;

ii) $t \mapsto f(t, 0)$;

iii) $t \mapsto f(t, \pi)$.

Exercise 2. (4 points)

Prove the following statements:

i) Let X_1 and X_2 denote the principal curvature directions on a surface at a given point and assume that the corresponding curvatures are not equal, i.e. $\kappa_1 \neq \kappa_2$. Then X_1 and X_2 are orthogonal.

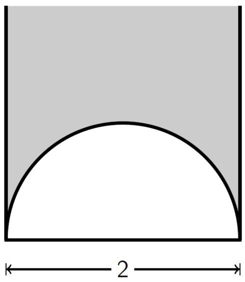
ii) There is no asymptotic line passing through an elliptic point[1].

Exercise 3. (4 points)

Let $r(t) = e^t$ and $h(t) = \int_0^t \sqrt{1 - e^{2x}}dx$. The resulting surface of revolution is called a *pseudosphere*. Determine the asymptotic lines and the curvature lines for the pseudosphere. Show that the asymptotic lines approach the limit circle ($t = 0$) tangentially for $t \to 0$.

Exercise 4. (4 points)

The hyperbolic space of dimension 2 can be modeled via the upper half plane $\mathbb{H}^2 = \{(u, v) \in \mathbb{R}^2 \mid v > 0\}$ equipped with the metric $g(u, v) = \frac{1}{v^2} I_2$, with I_2 the identity matrix of dimension 2. Consider the following geodesic triangle[2] in $\mathbb{H}^2$ whose vertices are lying all at infinity:



Determine its area by integration.

[1] Such a point has Gaussian curvature larger zero.

[2] The triangle is given by the part colored in gray.