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WiSe 2024/25

Version: 1

Differential Geometry III – Homework 5

Submission: 29. November 2024, until 8:15 am (start of the exercise class).

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1. Exercise (3 points)

Let $F = (f_1, f_2, f_3)$ be a conformal parameterization of a surface and $\varphi_i := (f_i)_u - \mathrm{i}(f_i)_v$ for $i = 1, 2, 3$. Complete the lecture from 5th November by proving the following statements:

i) It holds $\langle \Delta F, N \rangle = 2\lambda H$, where H is the mean curvature, N the normal vector and $g = \lambda \cdot \mathrm{id}$ is the metric induced by F.

ii) The surface F is regular if and only if $\varphi_1 \overline{\varphi_1} + \varphi_2 \overline{\varphi_2} + \varphi_3 \overline{\varphi_3} \neq 0$ everywhere.

iii) It holds $\varphi_1^2 + \varphi_2^2 + \varphi_3^2 = 0$ for $\varphi_1 = \frac{1}{2} f(1 - g^2)$, $\varphi_2 = \frac{\mathrm{i}}{2} f(1 + g^2)$ and $\varphi_3 = f \cdot g$.

2. Exercise (5 points)

Three triply periodic minimal from the lecture, Schwarz P, Schwarz D and the Gyroid, can each be nicely approximated by exactly one of the following implicit surfaces

$$\sin x + \sin y + \sin z = 0, \tag{1}$$

$$\sin x \sin y \sin z + \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z = 0, \tag{2}$$

$$\sin x \cos y + \sin y \cos z + \sin z \cos x = 0. \tag{3}$$

i) Find all straight lines and all planar geodesics contained in the Schwarz P, Schwarz D and the Gyroid, see Figure 1. Sketch those that you have found.

ii) Determine which equation corresponds to which surface and give solid arguments for it. A glance at Figure 1 might help.

Hint: Add the origin and the unit vectors e_x, e_y, e_z to the pictures in Figure 1, such that the orientation matches to the implicit equation. Then, examine translation symmetries and use the theorem from lecture on 19th November.

Please turn the page!

Total: 8

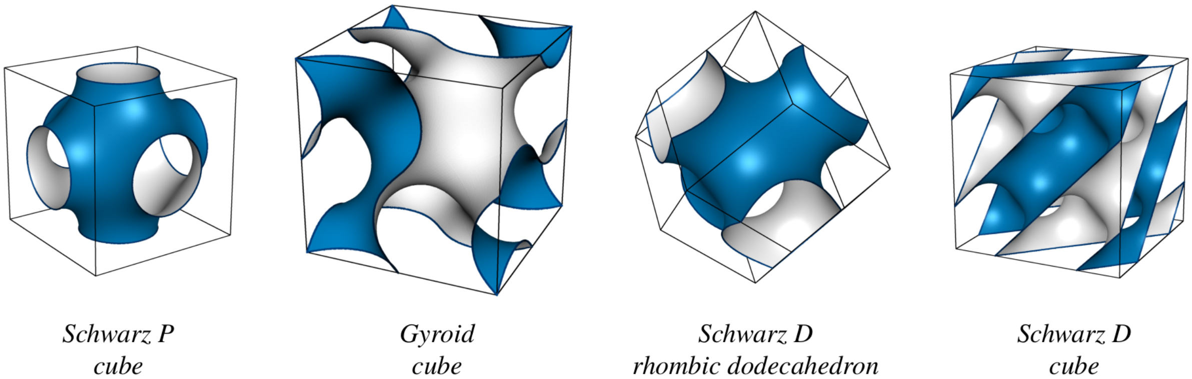

Figure 1: Translational units for triply periodic minimal surfaces. The picture is taken from "Discrete Gyroid Surface", Reitebuch, Skrodzki, Polthier (2019).