Freie Universität Berlin
Institut für Mathematik
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Version: 1

Differential Geometry III – Homework 4

Submission: 22. November 2024, until 8:15 am (start of the exercise class).

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1. Exercise (2 points)

How does the minimal surface F defined by the Weierstrass representation with the functions f and g change under the transform $(f, g) \mapsto (\mathrm{e}^{\mathrm{i}\theta} f, \mathrm{e}^{-\mathrm{i}\theta} g)$ with $\theta \in \mathbb{R}$?

2. Exercise (3 points)

Let F resp. H be the real resp. imaginary part of $\int_0^z \Phi(w)\,\mathrm{d}w$, where $\Phi = (\varphi_1, \varphi_2, \varphi_3)$ is the Weierstrass function with domain $\Omega \subseteq \mathbb{C}$, and consider the associated family

$$F^\theta := F\cos\theta + H\sin\theta \qquad \text{with } \theta \in [0, 2\pi).$$

i) How to choose $\tilde{f}$ and $\tilde{g}$ in the Weierstrass representation such that $\tilde{F} = F^\theta$?

ii) Prove, that the normal vector of the surface F^θ and the metric induced by F^θ on the domain Ω are both independent of θ.

3. Exercise (3 points)

Consider the following parameterizations for the Catenoid resp. the Helicoid

$$\mathbb{R}^2 \ni (u, v) \mapsto \begin{pmatrix} \cos v \cosh u \\ \sin v \cosh u \\ u \end{pmatrix} \qquad \text{resp.} \qquad \mathbb{R}^2 \ni (t, v) \mapsto \begin{pmatrix} t \sin v \\ -t\cos v \\ v \end{pmatrix}. \tag{1}$$

i) Are the parameterizations in (1) conformal? If not, find a suitable parameter transformation to obtain a conformal parameterization.

ii) Show that the Catenoid and the Helicoid are a pair of conjugate harmonic surfaces F and H.

iii) Draw the Catenoid and Helicoid next to each other and highlight the parameter lines that correspond to each other.

Total: 8