

## Bonus Exercise Sheet

Submission: 25.07.2023, 10:15 AM

Note: This sheet is not mandatory and it is supposed to gather missing points. It recovers some exercises from the first half of the course.

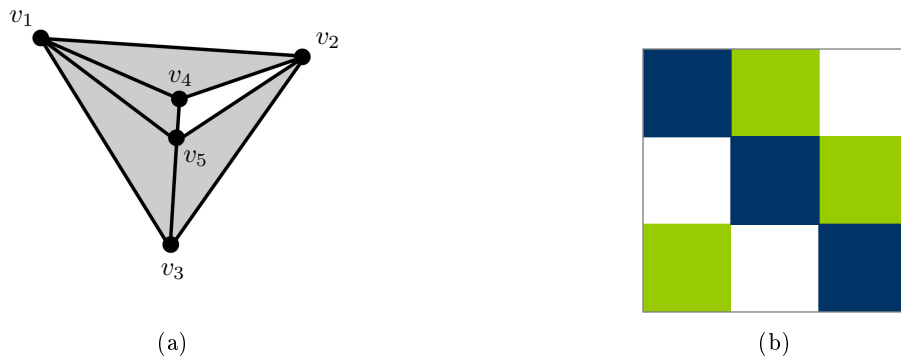


Figure 1: Left: Drawing of simplicial complex. Right: Image for texturing.

**Exercise 1.** (4 points) Let  $P = \{P_0, \dots, P_3\} \subset \mathbb{R}^2$  be a set of control points given by

$$P_0 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, P_2 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \text{ and } P_3 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Give an explicit representation of the *Bézier curve* given by the set of control points. Calculate new vertices arising from the *algorithm of de Casteljau* for  $t = 0.5$ .

**Exercise 2.** (4 points) Consider the simplicial complex illustrated in Figure 1a, where shaded regions depict 2-faces. Write down the simplicial complex and determine  $\text{link}(\{v_4, v_5\})$  and  $\text{star}(\{v_4\})$ . Give a drawing of an example for a simplicial complex having Euler characteristic  $-1$ .

**Exercise 3.** (4 points) Consider the set of control points  $S^0 = P$ , with  $P$  from Exercise 1. Determine the first two subdivision steps  $S^1$  and  $S^2$  starting with  $S^0$  using Chaikin's corner cutting scheme explicitly.

**Exercise 4.** (4 points) Let  $p_0 = (1, 0, 0)$ ,  $p_1 = (0, 2, 0)$ ,  $p_2 = (0, 0, 1)$  be vertices of a triangle  $\mathcal{T}$  in  $\mathbb{R}^3$ ,  $p = \frac{1}{2}(1, 2, 0)$  a point in  $\mathcal{T}$ , and an image with image domain  $D_I = [0, 3] \times [0, 3]$  shown in Figure 1b. Provide an example for a texture map  $t : \mathcal{T} \rightarrow D_T = [0, 1]^2$  with  $D_T$  is normalized texture domain and a map  $L : D_T \rightarrow D_I$  mapping it to the image domain. Sketch your maps and the result of the texture remapped to the triangle. Which color is assigned to  $p$ , i.e. what is  $I \circ L \circ t(p)$ ?

**Exercise 5.** (4 points) Consider the 1D image consisting of eight pixels given by pixel values

$$C = [c_0, \dots, c_7] = [2, 4, 0, 1, 1, -2, 4, 7].$$

The values  $c_i$  determine a function  $f = \sum c_i \Phi_i^3 \in V^3$  w.r.t. the box basis  $\mathcal{B} = \{\Phi_i^3\}$  of  $V^3$ ,  $i \in \{0, \dots, 7\}$ . Sketch  $f$  and determine the wavelet transformation of  $f$ .