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Scientific Visualization
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Freie Universität Berlin

Bonus Exercise Sheet

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Note: This sheet is not mandatory and it is supposed to gather missing points. It recovers some exercises from the first half of the course.

Figure 1: Left: Drawing of simplicial complex. Right: Image for texturing.

Exercise 1. (4 points) Let $P = \{P_0, \ldots, P_3\} \subset \mathbb{R}^2$ be a set of control points given by

$$P_0 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, P_2 = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \text{ and } P_3 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$

Give an explicit representation of the *Bézier curve* given by the set of control points. Calculate new vertices arising from the *algorithm of de Casteljau* for $t = 0.5$.

Exercise 2. (4 points) Consider the simplicial complex illustrated in Figure 1a, where shaded regions depict 2-faces. Write down the simplicial complex and determine $\text{link}(\{v_4, v_5\})$ and $\text{star}(\{v_4\})$.
Give a drawing of an example for a simplicial complex having Euler characteristic -1.

Exercise 3. (4 points) Consider the set of control points $S^0 = P$, with P from Exercise 1. Determine the first two subdivision steps S^1 and S^2 starting with S^0 using Chaikin's corner cutting scheme explicitly.

Exercise 4. (4 points) Let $p_0 = (1, 0, 0)$, $p_1 = (0, 2, 0)$, $p_2 = (0, 0, 1)$ be vertices of a triangle $\mathcal{T}$ in $\mathbb{R}^3$, $p = \frac{1}{2}(1, 2, 0)$ a point in $\mathcal{T}$, and an image with image domain $D_I = [0, 3] \times [0, 3]$ shown in Figure 1b. Provide an example for a texture map $t : \mathcal{T} \to D_T = [0, 1]^2$ with D_T is normalized texture domain and a map $L : D_T \to D_I$ mapping it to the image domain. Sketch your maps and the result of the texture remapped to the triangle. Which color is assigned to p, i.e. what is $I \circ L \circ t(p)$?

Exercise 5. (4 points) Consider the 1D image consisting of eight pixels given by pixel values

$$C = [c_0, \ldots, c_7] = [2, 4, 0, 1, 1, -2, 4, 7].$$

The values c_i determine a function $f = \sum c_i \Phi_i^3 \in V^3$ w.r.t. the box basis $\mathcal{B} = \{\Phi_i^3\}$ of V^3, $i \in \{0, \ldots, 7\}$. Sketch f and determine the wavelet transformation of f.