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Scientific Visualization
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Freie Universität Berlin

Exercise Sheet 9

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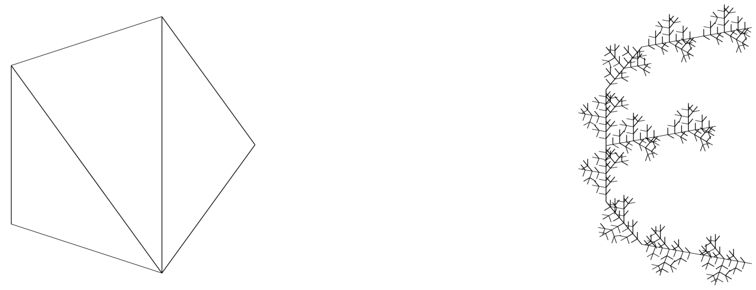

Figure 1: Left shows a representation of a regular pentagon with two additional edges and right a representation of the tutors' initial using L-systems.

Exercise 1. (8 points)
Let $v, v' : \mathbb{R}^2 \to \mathbb{R}^2$ be vector fields and $f : \mathbb{R}^2 \to \mathbb{R}$ a scalar field. Both are supposed to be twice continuously differentiable[1].

i) Show that $\operatorname{curl}(\operatorname{grad}(f)) = 0$. What can you tell about $\operatorname{div}(\operatorname{curl}(v))$?

ii) Show whether $f : \mathbb{R}^2 \setminus \{0\} \to \mathbb{R}$, $(x, y) \mapsto \left(x^2 + y^2\right)^{-\frac{1}{2}}$ is harmonic[2]. Describe and sketch how the gradient of f behaves and looks as a vector field.

iii) Suppose $\operatorname{div} v = \operatorname{div} v' = 0$. Do $v + v'$ and $v \odot v'$ necessarily have zero divergence[3]?

iv) Show that $\operatorname{div}(fv) = \operatorname{grad}(f) \cdot v + f \operatorname{div}(v)$ with $\cdot$ denoting the scalar product.

Exercise 2. (8 points) Solve the following tasks using L-systems[4].

i) Use the alphabet $\{F, f, +, -, [,]\}$ as mentioned in the lecture and an appropriate angle to find a word describing the pattern in Figure 1 on the left, which is a regular[5] pentagon with two additional edges.

ii) Let a turtle draw your first initial[6] in a creative way, like in Figure 1 on the right. Therefore, provide the used alphabet, replacement rules, axiom, angle, and number of iterations. Either attach an image to your solutions or send it via mail with your submission.

[1] Hint: Schwarz's theorem about the equality of mixed partials might be helpful.
[2] A twice differentiable function $f : \mathbb{R}^2 \to \mathbb{R}$ is *harmonic* if $\operatorname{div}(\operatorname{grad}(f)) = 0$.
[3] Here $\odot$ denotes the *Hadamard product*, i.e. for two matrices A, B of same dimension $m \times n$ the product is an $m \times n$ matrix with entries $(A \odot B)_{ij} = (A)_{ij}(B)_{ij}$. In our case an n-vector could be considered an $n \times 1$ matrix.
[4] You can find a project dealing with L-systems in JavaView: File - New - Project - L-System.
[5] All the edges have the same length and the inner angles are of the same size for the regular pentagon itself.
[6] One example per group suffices.