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Scientific Visualization
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Exercise Sheet 8

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Exercise 1. (8 points) Let $\mathcal{M}$ be a simplicial surface and define S as

$$S \coloneqq \left\{ f : \mathcal{M} \to \mathbb{R} \mid f \text{ is (affine) linear on each } \sigma \in \mathcal{M} \text{ and } f \in C^0(\mathcal{M}) \right\}.$$

Show that S is a real vector space (equipped with pointwise addition and scalar multiplication).

Exercise 2. (8 points) Consider the following triangulation with vertices

$$P_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_2 = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, P_3 = \frac{1}{2}\begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix}, P_4 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, P_5 = -\frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \text{ and } P_6 = \frac{1}{2}\begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix},$$

and triangles $T_i = [P_0, P_i, P_{i+1}]$ for $i = 1, \ldots, 5$ and $T_6 = [P_0, P_6, P_1]$. Let further $u \in S$ be a continuous, piecewise (affine) linear function with values $u_i \coloneqq u(P_i)$ given by

$$u_0 = 1, u_1 = 1, u_2 = 1, u_3 = \frac{1}{2}, u_4 = \frac{1}{3}, u_5 = 2, \text{ and } u_6 = -3.$$

i) Determine the gradient field ∇u and sketch it.

ii) For the triangle $T_0 = [P_0, P_1, P_2]$ let $w \in S$ be a function with $w_1 = 1, w_2 = -1$, and gradient $\nabla w_{|T_0} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$. Determine w_0.