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Version: 1

Scientific Visualization
Summer Semester 2023
Freie Universität Berlin

Exercise Sheet 7

Submission: 27.06.2023, 10:15 AM

Exercise 1. (3 points) Show that on $\mathbb{R}^3 \setminus \{0\}$ an equivalence relation is given by

$$v \sim v' \iff \exists \lambda \in \mathbb{R} \setminus \{0\} : v = \lambda \cdot v'.$$

Exercise 2. (8 points)

i) Given the rotation R_φ around the angle $\varphi \in [0, 2\pi)$, the scaling S_λ by $\lambda \in \mathbb{R} \setminus \{0\}$, and the translation T_t by $t \in \mathbb{R}^2$ as (affine) maps $\mathbb{R}^2 \to \mathbb{R}^2$. Determine the corresponding linear maps $\mathbb{R}^3 \to \mathbb{R}^3$ and R_φ^{-1}, S_λ^{-1}, T_t^{-1}, and $(R_\varphi T_t)^{-1}$ in terms of R_φ, T_t, and S_λ.

ii) Let $P = \{P_1, P_2, P_3, P_4\} \subset \mathbb{R}^2$ be given by

$$P_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, P_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ and } P_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Determine an affine map $A : \mathbb{R}^2 \to \mathbb{R}^2$ such that $P_i \mapsto A(P_i) = Q_i$ for $i \in \{1, 2, 3, 4\}$ and

$$Q_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, Q_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, Q_3 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \text{ and } Q_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

If possible determine A^{-1}.

Exercise 3. (5 points) Consider the simplicial surface defined by the set of points

$$P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, P_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, P_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \text{ and } P_4 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

with triangles $\{P_1, P_3, P_2\}$, $\{P_1, P_4, P_3\}$, and $\{P_1, P_2, P_4\}$. Determine the error quadric Q_1 at point P_1 as defined in the algorithm by Garland and Heckbert.