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Scientific Visualization
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Freie Universität Berlin

Exercise Sheet 6

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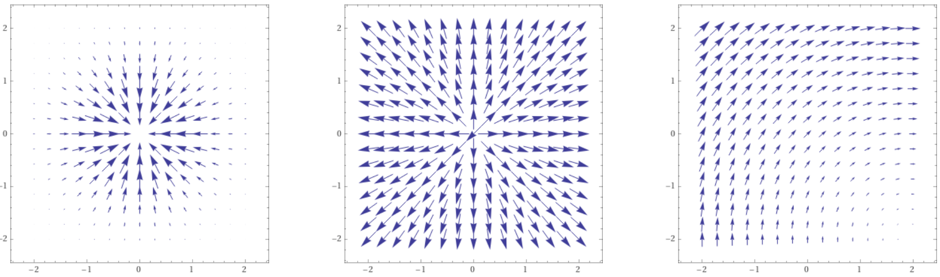
Exercise 1. (6 points) Consider the following three representations of three vector fields in $\mathbb{R}^2$:



i) Find explicit representations on the open square U approximating the three shown vector fields.

ii) Try to find potential functions[1] $f \in C^1(U)$, $i = 1, 2, 3$, whose gradient[2] fields gradf_i look as in the figures above, or explain if and why such a function cannot exist.

Exercise 2. (5 points) Let $D_I = [-2, 2]^2 \subset \mathbb{R}^2$ be an image domain and $v : \mathbb{R}^2 \to \mathbb{R}^2$, $(x, y) \mapsto (-y, x)$ a vector field. Further let $\varepsilon = 1$, $x_0 = (1, 0)$, and $L = 2$.

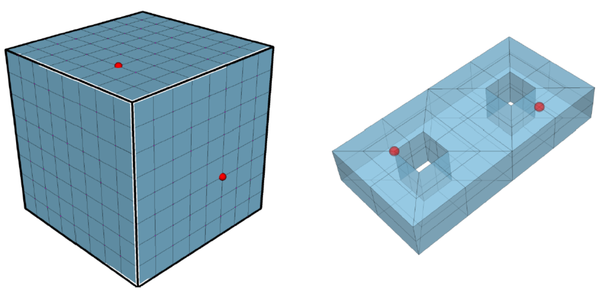
i) Calculate the values $\{\gamma_i\}$ for $i \in \{0, \dots, L\}$ approximating the integral curve $\gamma : [0, 1] \to \mathbb{R}^2$ using the Euler method w.r.t. step size ε, starting point x_0, and vector field v.

ii) Determine the LIC color values $I(\gamma_0)$ and $I(\gamma_1)$ w.r.t. L using some white noise[3].

Exercise 3. (5 points)

i) For the cube shown below (left image) find and sketch two straightest discrete geodesics connecting the two highlighted points, one of them being a shortest, the other one not being a shortest geodesic. Justify your solution.

ii) Consider the quadrangular surface of genus 2 (right image). Find and sketch three straightest geodesics connecting the two highlighted points.



[1]Hint: Let $f : U \to \mathbb{R}^n$, $U \subset \mathbb{R}^n$ open, with partial derivatives $\frac{\partial f}{\partial x_i}$, $\frac{\partial f}{\partial x_j}$, and further $\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} f$ exists and is continuous. Then $\frac{\partial}{\partial x_j}\frac{\partial}{\partial x_i} f$ exists and $\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j} f = \frac{\partial}{\partial x_j}\frac{\partial}{\partial x_i} f$, cf. Theodor Bröcker - Analysis II.

[2]If $f : U \subset \mathbb{R}^n \to \mathbb{R}$ is differentiable in u then the vector $\text{grad}(f) = \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_n}\right)$ is called *gradient* of f at point u.

[3]Here you might consider random values from a noise function $N : D_I \to C$, with C a color space of gray scales distributed uniformly. Fun question: How can you simulate to draw uniformly from $\{0, \dots, 255\}$ with the least sum Σ of natural objects and attempts? For instance throwing a dice once would give you a number from 1 to 6, i.e. $\Sigma = 2$.