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Scientific Visualization
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Exercise Sheet 5

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Exercise 1. (4 points)

i) Show that the spaces of *pixel functions* $V^j, j \in \mathbb{N}_0$, are nested, i.e. $V^0 \subset V^1 \subset \dots$.

ii) Let W^k denote the k-th *Haar space*, $k \in \{0, \dots, j-1\}$. Show the following decomposition of V^j:

$$\begin{aligned} V^j &= V^{j-1} \oplus W^{j-1} \\ &= V^0 \oplus W^0 \oplus W^1 \oplus \dots \oplus W^{j-1}. \end{aligned}$$

Exercise 2. (12 points) Consider the 1D image consisting of eight pixels given by the pixel value array

$$C = [c_0, \dots, c_7] = [4, 8, 6, 2, -2, -1, 0, 3]\,.$$

The values c_i determine a function $f = \sum c_i \Phi_i^3 \in V^3$ w.r.t. the box basis[1] $\mathcal{B} = \left\{\Phi_i^3\right\}$ of V^3, $i \in \{0, \dots, 7\}$.

i) Express f in terms of the basis $\mathcal{H} = \left\{\Phi_0^0\right\} \cup \left\{\Psi_i^j\right\}$, $j \in \{0, 1, 2\}$, $i \in \left\{0, \dots, 2^j - 1\right\}$ for the decomposition $V^3 = V^0 \oplus W^0 \oplus W^1 \oplus W^2$, where for each j, $\left\{\Psi_i^j\right\}$ is the Haar basis for W^j.

ii) For the decomposition $f = v + w_0 + w_1 + w_2$ just derived, sketch the partial sums $v + \dots + w_k$ for $k \in \{0, 1, 2\}$.

iii) Determine the wavelet compression $\hat{f}$ of f such that $\|f - \hat{f}\|_2 < 2$. What are the coefficients of $\hat{f}$ w.r.t. the basis $\mathcal{H}$?

iv) Express $\hat{f}$ in terms of the box basis $\mathcal{B}$ and sketch it.

[1] Here we use letters Φ and Ψ denoting the box basis functions and Haar basis functions according to the script.