

Exercise Sheet 2

Submission: 09.05.2023, 10:15 AM

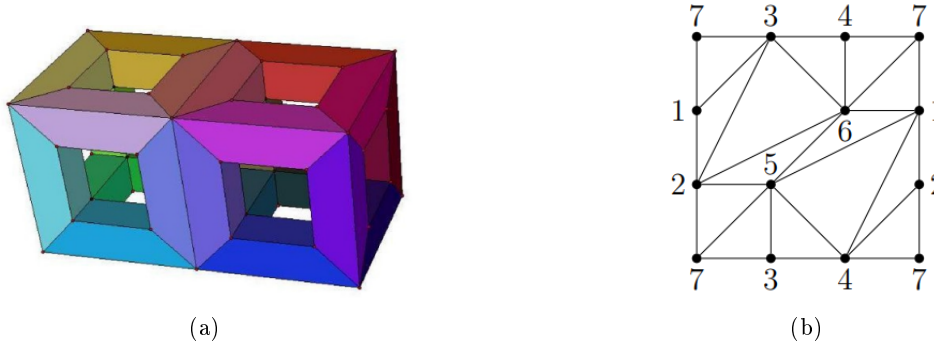


Figure 1: Left: Quadrilateral surface. Right: Drawing of *Császár torus*.

Exercise 1. (12 points)

- i) Determine the genus of a closed simplicial surface with 20 triangles and 12 vertices.
- ii) Sketch¹ two nonisomorphic examples of a closed simplicial surface with 20 triangles and 12 vertices which are both not the icosahedron. Why are your examples nonisomorphic?
- iii) How many edges does a simplicial double torus with 1200 vertices have?
- iv) Let Q be a quadrilateral surface (see Figure 1a for an example) and v, e, f, g , and χ denote the number of vertices, number of edges, number of faces, genus, and Euler characteristic of Q .
 - a) Show that $\chi = v - e + f$ still holds.
 - b) Determine v, e, f, g , and χ for the example shown in Figure 1a.
- v) Give three examples for simplicial complexes having the same Euler characteristic but which are not simplicially isomorphic². Justify your choice.

Exercise 2. (4 points) The *Császár torus*³ is a two-dimensional simplicial complex consisting of 7 vertices, 21 edges, and 14 triangles, cf. Figure 1b. Vertices labeled with the same index are identified and the edges are identified accordingly. Determine

- $\text{star}([2]), \text{star}([3,4]), \text{star}([1,5,6]),$
- $\text{link}([2]), \text{link}([3,4]),$ and $\text{link}([1,5,6]).$

Please use the definition of a star stated in the script.

¹Think of an appropriate representation.

²They are not isomorphic as simplicial complexes.

³A model of the *Császár torus* can be found in JavaView, i.e. in File - Open - JavaView Models in the category Polytope.