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Scientific Visualization
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Freie Universität Berlin

Exercise Sheet 1

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Exercise 1. (3 points) Let $\mathcal{P} = \{P_0, \ldots, P_2\} \subset \mathbb{R}^2$ be a set of control points given by

$$P_0 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, P_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \text{ and } P_2 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}.$$

Determine all resulting control points using the *de Casteljau algorithm* for $t = 0.5$. Determine b(t) explicitly. Represent the given control points, the resulting control points, and b(t) graphically.

Exercise 2. (3 points) Let m_0 and m_1 be two tangent directions belonging to the points P_0 and P_1 given by

$$m_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ and } m_1 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, P_0 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \text{ and } P_1 = \begin{pmatrix} 4 \\ 4 \end{pmatrix}.$$

Determine a C^1 cubic polynomial passing through the given points with the given tangent directions. Illustrate your results.

Exercise 3. (3+2=5 points) Show the following properties of the *Bernstein polynomials* with $n \in \mathbb{N}_0$ and $i \in [n]_0$:

i) $B_i^n(t)$ has exactly one maximum in $[0, 1]$ for $n > 0$;

ii) $B_i^n(t) = \frac{i+1}{n+1} B_{i+1}^{n+1}(t) + \frac{n+1-i}{n+1} B_i^{n+1}(t)$.

Exercise 4. (5 points) Represent your initials with combined Bézier curves using postscript[1] based on the code presented in the lecture[2]. An example is shown in Figure 1. Send in an executable ps-file called 01-Name0Name1.ps (one choice of initials per group suffices).

Figure 1: An illustrative example.

[1] An interpreter for postscript can be found on https://www.ghostscript.com/index.html.
[2] Additional information can be found in the lecture notes and script uploaded on the course page.