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Version: 1

Differential Geometry I
Winter Semester 2023/2024
Freie Universität Berlin

Bonus Sheet

Submission: 20.02.2024, 12:15 PM

Note: This sheet contains 18 bonus points.

Exercise 1. (4 points)
Determine the curvature κ and torsion τ of the curve $t \mapsto (\sin(t), -t, \cos(t))$.

Exercise 2. (5 points)
For a profile curve mapping $u \mapsto (r(u), h(u))$ let

$$f : [a,b] \times [0, 2\pi) \to \mathbb{R}^3$$
$$(u,v) \mapsto \begin{pmatrix} \cos(v) r(u) \\ \sin(v) r(u) \\ h(u) \end{pmatrix}$$

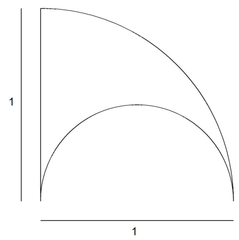
parametrize a *surface of revolution*. Determine for f the differential Df, surface normal N, metric g, second fundamental form b, shape operator S, principal curvatures κ_1, κ_2, Gaussian curvature K, and mean curvature H.

Exercise 3. (3 points)
Is there a surface $f : \Omega \to \mathbb{R}^3$ with the following first fundamental form g and shape operator S, i.e.

$$g = \begin{pmatrix} 1 & 0 \\ 0 & \cos^2(u) \end{pmatrix} \text{ and } S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}?$$

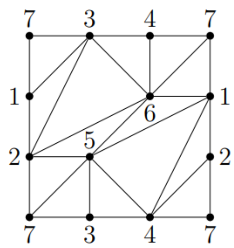
Justify your solution.

Exercise 4. (3 points)
The hyperbolic space of dimension 2 can be modeled via the upper half plane $\mathbb{H}^2 = \left\{(u,v) \in \mathbb{R}^2 \mid v > 0\right\}$ equipped with the metric $g(u,v) = \frac{1}{v^2} I_2$, with I_2 the identity matrix of dimension 2. Consider the following geodesic triangle[1]:



Determine its area by integration.

Exercise 5. (3 points)
Determine star([1,6]), link([7]), and the total Gauß curvature of the *Császár torus*[2] shown below.



[1]The triangle is enclosed by the three arcs.
[2]A model of the *Császár torus* can be found in JavaView, i.e. in File - Open - JavaView Models in the category Polytope.