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Version: 1

Differential Geometry I
Winter Semester 2023/2024
Freie Universität Berlin

Exercise Sheet 12

Submission: 13.02.2024, 12:15 PM (start of lecture)

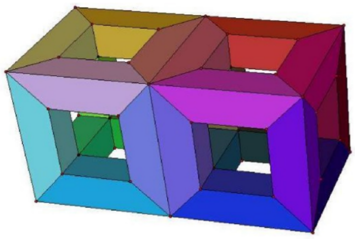

Figure 1: Quadrilateral surface.

Exercise 1. (4 points)
Let p be an inner point of a simplicial regular[1] surface S.

i) Determine the number of triangles incident to p such that the discrete Gauss curvature K in p is equal to $\frac{2\pi}{3}$, 0, or $-\frac{2\pi}{3}$ resp. Illustrate your results.

ii) Determine the number of triangles incident to p such that $K(p) = 42\pi$ resp. $K(p) = -42\pi$.

iii) Determine the discrete Gauss curvature for the surface depicted in Figure 1 where all angles are equal to $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$ as indicated in the figure.

Exercise 2. (4 points)
Consider the triangle Δ given by the three vertices $p_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $p_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$, and $p_3 = \begin{pmatrix} 2 \\ -1/2 \end{pmatrix}$.

i) Determine the *hat functions* φ_i, $i \in \{1, 2, 3\}$, introduced in the lecture on Δ explicitly.

ii) Illustrate your results.

iii) Find the linear combination of a constant function on Δ.

iv) Find the barycenter.

Exercise 3. (8 points)
Let $\mathcal{M}$ be a simplicial surface and define S as

$$S := \left\{f : \mathcal{M} \to \mathbb{R} \mid f \text{ is (affine) linear on each } \sigma \in \mathcal{M} \text{ and } f \in C^0(\mathcal{M})\right\}.$$

Show that S is a real vector space (equipped with pointwise addition and scalar multiplication).

[1] All edges have the same length, and therefore, all angles are equal, too.