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Version: 2

Differential Geometry I
Winter Semester 2023/2024
Freie Universität Berlin

Exercise Sheet 9

Submission: 16.01.2024, 12:15 PM (start of lecture)

Note: This sheet contains 1 bonus point.

Exercise 1. (3 points)
Let M be a closed surface triangulated by a finite triangulation $\mathcal{T}$. For a vertex $p \in \mathcal{T}$ we define the discrete graph Gauss curvature K by

$$K(p) \coloneqq 6 - \operatorname{val}(p),$$

where $\operatorname{val}(p)$ denotes the valence of the vertex p, i.e. the number of adjacent edges. Show that the following version of a discrete Gauss-Bonnet theorem holds:

$$\int_M K \coloneqq \sum_{p \in \mathcal{T}} K(p) = 6\chi(M).$$

Exercise 2. (7 points)

i) Show that for a closed quadrilateral surface the Euler characteristic $\chi = v - e + f$ still holds, with v, e, and f the number of vertices, edges, and faces, respectively.

ii) Construct a quadrilateral torus T and illustrate your result.

iii) Count the vertices of T with positive, negative, and zero angle defect.

iv) Determine the Euler characteristic χ the total discrete Gaussian curvature of T.

Exercise 3. (4 points)
Consider a parametrized surface S with negative Gaussian curvature $K < 0$ everywhere. Prove the following statements:

i) At each point $q \in S$ there are two linearly independent asymptotic directions, i.e. two linearly independent tangent vectors $X, Y \in T_qS$ such that $b(X, X) = b(Y, Y) = 0$.

ii) S is a minimal surface if and only if the asymptotic directions in each point on S are perpendicular to each other.

Exercise 4. (3 points)
Show that the surface parametrized by $f(u, v) = \left(u, v, \log\left(\frac{\cos(v)}{\cos(u)}\right)\right)$, $\cos(u) \neq 0$ and $\frac{\cos(v)}{\cos(u)} > 0$, is a minimal surface.