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Version: 1

Differential Geometry I
Winter Semester 2023/2024
Freie Universität Berlin

Exercise Sheet 8

Submission: 09.01.2024, 12:15 PM (start of lecture)

Note: This sheet contains 8 points and therefore 3 bonus points.

Exercise 1. (3 points)
Which tiling by a regular (n, α)-gon has the same symmetries as the one shown in Figure 1? Provide the number of edges n and the corner angle α of the polygon and draw the tiling by the (n, α)-gon on top of Figure 1.

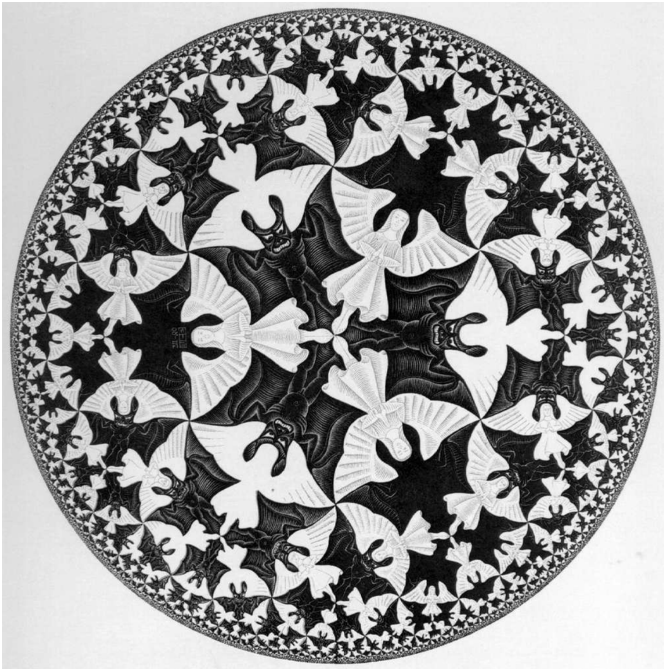

Figure 1: M. C. Escher's "Circle Limit IV", (c) 1997 Cordon Art. All rights reserved. Available from: researchgate, accessed 14 Dec, 2023.

Exercise 2. (8 points)
Consider the so-called *Cayley map*[1]

$$f_C : \hat{\mathbb{C}} \to \hat{\mathbb{C}}, \quad z \mapsto \frac{z-i}{z+i}.$$

i) Sketch the images under f_C of those lines which are parallel to the real or imaginary axis.

ii) Determine f_C^{-1}.

iii) Conclude that f_C is a bijection from the upper half-plane $\mathbb{H} := \{z \in \mathbb{C} \mid \mathrm{Im}(z) > 0\}$ to the unit disk $\mathbb{D} := \{w \in \mathbb{C} \mid\mid w \mid < 1\}$.

iv) Determine the coordinates of an arbitrary point (x, y) in the half-plane model that is mapped to the disk model.

Exercise 3. (0 points)
Enjoy your holidays and have a good start into the new year!

[1] A complex number $z \in \mathbb{C}$ can be written as $z = x + iy$ with $x = \mathrm{Re}(z) \in \mathbb{R}$ (real part of z) and $y = \mathrm{Im}(z) \in \mathbb{R}$ (imaginary part of z), and i the imaginary unit with property $i^2 = -1$. Further we set $\hat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ and for $z = x + iy \in \mathbb{C}$ we have $\mid z \mid := \sqrt{x^2 + y^2}$.