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Scientific Visualization – Homework 6

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1. Exercise (4 points)

1.) Show that the spaces of *pixel functions* V^j, $j \in \mathbb{N}_0$, are nested: $V^0 \subset V^1 \subset \dots$.

2.) Let W^k denote the k-th *Haar space*, $k \in \{0, \dots, j-1\}$. Show the following decomposition of V^j:

$$\begin{aligned} V^j &= V^{j-1} \oplus W^{j-1} \\ &= V^0 \oplus W^0 \oplus W^1 \oplus \dots \oplus W^{j-1}. \end{aligned}$$

2. Exercise (12 points)

Consider the 1D image consisting of eight pixels given by the pixel value array

$$C := [c_0, \dots, c_7] := [4, 8, 6, 2, -2, -1, 0, 3].$$

These values c_i determine a function $f := \sum c_i \Phi_i^3 \in V^3$ with respect to the box basis $\mathcal{B} := \{\Phi_i^3\}$ of V^3, $i \in \{0, \dots, 7\}$.

1.) Express f in terms of the orthonormal[I] basis $\mathcal{H} := \{\Phi_0^0\} \cup \{\Psi_i^j\}$, $j \in \{0, 1, 2\}$, $i \in \{0, \dots, 2^j - 1\}$ for the decomposition $V^3 = V^0 \oplus W^0 \oplus W^1 \oplus W^2$, where for each j, $\{\Psi_i^j\}$ is the (orthonormal) Haar basis for W^j.

2.) For the decomposition $f = v + w_0 + w_1 + w_2$ just derived, sketch the partial sums $v + \dots + w_k$ for $k \in \{0, 1, 2\}$.

3.) Determine the wavelet compression $\hat{f}$ of f such that $\left\| f - \hat{f} \right\|_2 < 2$. What are the coefficients of $\hat{f}$ with respect to the basis $\mathcal{H}$?

4.) Express $\hat{f}$ in terms of the box basis $\mathcal{B}$ and sketch it.

Total: 16

[I] I.e. with respect to the L^2-scalar product. You may think of C as a function in $L^2([0,1])$, piecewise constant on the intervals $[i/8, (i+1)/8)$, $i = 0, \dots, 7$.